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PHASE B - COMPLETION REPORT
NAS 8-11715 PART III

15 JANUARY 1965

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SECTION 1

SUMMARY

This report gives the second Phase (Phase B) results obtained on NAS 8-11715 contract, in the area of single parameter testing. The main objective of the study is to put into operation better ways of testing components during stage checkout. The expected gains include a reduction in component checkout time and improved component performance evaluation during checkout.

The result of the study through Phase B is positive. We can test linear active and passive systems that exhibit first and second order transfer functions. The savings include reduced checkout time, possible isolation of failures, and the ability to test with small signal amplitudes.

Many technical areas have been investigated with two techniques showing direct applicability to the solution of single parameter testing of linear systems.

These techniques are:

1. The growing exponential technique which utilizes a specified increasing exponential function as a probing signal.
2. A sampling technique developed from the study in opti-

mization techniques.

The first technique involving growing exponential signals has been tested experimentially on the analog computer. The results for testing systems exhibiting first and second order transfer functions is positive, in that parameter variations can be measured. The main disadvantage with this technique is the requirement that a nominal system is used to null against the actual system. As yet, limitations have not been encountered experimentally or mathematically in applying the method.

The second technique involving sampling has been considered mathematically. The experimental results from the growing exponential technique can be extrapolated to this sampling method to demonstrate its application in single parameter testing. The paramount gain is the reduction in the amount of hardware required for implementation.

What problems require further investigation? The growing exponential technique must be evaluated in a real system application and useful comparisons acquired to evaluate its practical competitive position.

The sampling technique must be evaluated in comparison to the growing exponential technique in a real system application.

SECTION 2

INTRODUCTION

The single parameter testing program was established to perform mathematical analysis on typical systems of varying complexity. The program will verify the applicability of single parameter testing to launch vehicle system checkout.

The general technical approach of the study was limited to systems or devices for which continuous transfer functions can be written and restricted to their linear regions. Primary emphasis was directed on the identification of changes in the terms which compose the transfer function.

The approach used was to:

1. Describe transfer functions and study the changes in behavior with incremental changes in its parameter, i.e., terms.
2. For each transfer function, investigate the measurability of performance degradation due to changes in the transfer function. One output of this task will be determining the feasibility of GO, NO-GO, decisions based on parameter testing.
3. Investigate possible theories of measuring single parameters to accomplish the measurement of incremental changes, and performance degradation due to transfer function changes.

The study is divided into three specific tasks:

Phase A: Simple first, second, and third order linear passive networks whose transfer functions resemble those of useful systems, were to be selected for detailed investigation. The results will be used to extend the testing to higher order systems.

Phase B: The investigation and selection of criteria described in Phase A. This is to include the linear active networks.

Phase C: With the guidance and approval of the NASA technical representative, an actual subsystem will be chosen for analysis. The transfer function will be derived, and the techniques developed in Phase A and B are to be applied to the subsystem.

SECTION 3

ACTIVE NETWORKS

3.1 REVIEW OF GROWING EXPONENTIAL TESTING

The first phase report (64 SIMAR 59/2) of this study, in the area of Single Parameter Testing, gave a comprehensive description of a testing scheme applying growing exponential signals. In order to bring the basic concept into focus a quick review is in order. The basic scheme consists of a specialized signal generator, the transfer function of interest, a nominal equivalent transfer function, a specialized set of filters, and a linear estimator. The overall idea is to generate a set of signals matched to a transfer function that represents a particular part of a system transfer function. This particular part is the first partial derivative of the system transfer function with respect to a parameter. Considered in strict mathematical terms, it is the first term in a Taylor Series Expansion of the system function when the independent variable is the parameter.

The discussion thus far is the heart of the agument of testing with growing exponential signals. Conceptionally, this particular matching of a partial derivative may be considered similar to a simple multiplication of an ordinary complex number by its conjugate. In both cases the invariant property of the squared function or quadratic offers mathematical simplicity. This is a difficult concept to grasp by a detailed mathematical description, therefore, as an example, change the specialized signal generator into a coordinate generator. Each particular component out of the signal generator represents a coordinate vector and the composite signal an entire coordinate system. Each coordinate is constructed to represent a change in a particular parameter, so each may be considered a vector proportional to a unit parameter change. With this description, clearly the generated coordinate system is related to the first partial derivatives as mentioned.

Now for a moment, consider a system transfer function with relation to future time. If the question of prediction is considered, clearly no ordinary system can predict what state it will exist in at some specified time without careful evaluation of a forcing function up to that particular time. If a signal from a sinusodial generator is known to be generated at a specific time, clearly, prediction only requires a knowledge of the signal amplitude now and the specific interval of time. This concept should clear up any doubt now in regards to the term matching. If the specialized generator is considered as the signal, all that is required is the initial amplitude and the time interval to allow prediction. Hence, if the coordinate system mentioned above is used by equivalently reversing the time parameter, the concept of matching has been completed.

In an overall view, the matched signal coordinate system looks exactly like an impulse response of the partial derivative system reversed in time. By using this as an input signal to the system transfer function an output signal will result with amplitudes assigned to each member of the coordinate system. With no parameter change, theoretically all values out of the total system would be zero except for the effect of $H_o(s)$ on each coordinate. That is, the Taylor Series expansion allows the system to be equivalent represented by $H_o(s)$ and partial derivative systems in parallel. Hence, each partial derivative system will attentuate the coordinates to zero and all that would be retained is a transformed input signal coordinate system. That is $H_o(s)\ \Phi(s)$, where $\Phi(s)$ is the input signal coordinate system.

Now the remaining parts of the scheme may be incorporated into the picture. If an equivalent nominal system transfer function is included in parallel and the same input signal is applied, a difference will allow removal of the undesired transformed input. Consequently, with no parameter change a null or zero output will exist. The filters can be added to the scheme in a serial fashion. That is, filter one will extract Φ_1 and all the rest of the coordinates, filter two will extract Φ_2 and all higher coordinates, but not Φ_1 which will be attenuated. This process can be used to extract signals up to Φ_n in a serial fashion. The filter outputs can then be scaled in a proper fashion by an estimator based on a linear regression relation and the output sampled at the time a value is known ($t = t_1$). The result can be an estimate of how much a parameter changed at some particular instant of time if the mathematics is properly carried through. If the sampling instant is conveniently chosen at ($t = 0$) and all scale factors properly carried through the mathematics, the result will be a linear function of the changing parameter.

A detailed mathematical discussion of this growing exponential scheme is contained in the first phase report of this study, and the interested reader should refer all detailed question to the detailed mathematics.

One term that has not been clearly stated is growing exponential signals. The term clearly comes from the form of the specialized generator output and is tied into the fact that linear system impulse response functions are normally decaying exponential functions. Consequently, partial derivative system functions will also be decaying exponential functions, and the matching coordinate system will consist of the exponentials with time reversed. Hence, the matching signal generator puts out growing exponential signals.

3.2 ACTIVE NETWORKS INVESTIGATED

The technique normally applied to analyzing an active network consists of picking a region of operation and an appropriate model that describes the operation over a specific dynamic range. The model then depends on the physical processes and the ultimate answers that are desired from the model. Since an accurate functional model can abstract the internal operation of the physical process only over a dynamic range, limits must be specified for the validity of the parameters in the functional model.

Now the concept of a small signal model can be clearly stated. The dynamic range is a small signal specified in terms of input-terminal signal condition, and the model parameters are the network parameters. This in turn allows the discussion of a linear active network model without describing the complex physical process. Thus, it allows the non-linearities in the physical process to be considered outside of the dynamic range and detailed questions regarding these non-linearities must be referred to another model.

Within this framework of a limited dynamic range for a particular model, all of the concepts of linear system analysis can be applied. That is, linear differential equations and corresponding Laplace and Fourier transforms can be used to evaluate the state of the model. At this point, questions regarding the existence of a transform, stability properties, system bandwidth, available output power and measureability can be considered. In order to limit the scope of the general system problem a specific interpretation must be considered.

The interpretation of an Active System used in this study is in reference to a device that exhibits a unilateral transfer characteristic. A simple amplifier is the most common example exhibiting this unilateral characteristic. Consequently, an amplifier circuit will be evaluated in terms of the proposed method of single parameter testing.

A number of amplifier circuits will be considered. One amplifier circuit to be investigated is shown schematically in Figure 3-1 along with the appropriate equivalent circuit or small signal model.

The transfer function relating input to output is derived in Section 3.2.1 as

$$\frac{e_o}{e_{in}} = \frac{S\, R_i\, \mu}{(R_g + R_i)\left[(1 + r_p/R_L + r_p/R_0)\, S + \dfrac{R_L + r_p}{C\, R_0 R_L}\right]}$$

or in standard form

$$\frac{e_o}{e_{in}} = \frac{K\, S}{d_1 S + d_0}$$

r_p = Tube dynamic plate resistance

μ = Tube amplification ratio

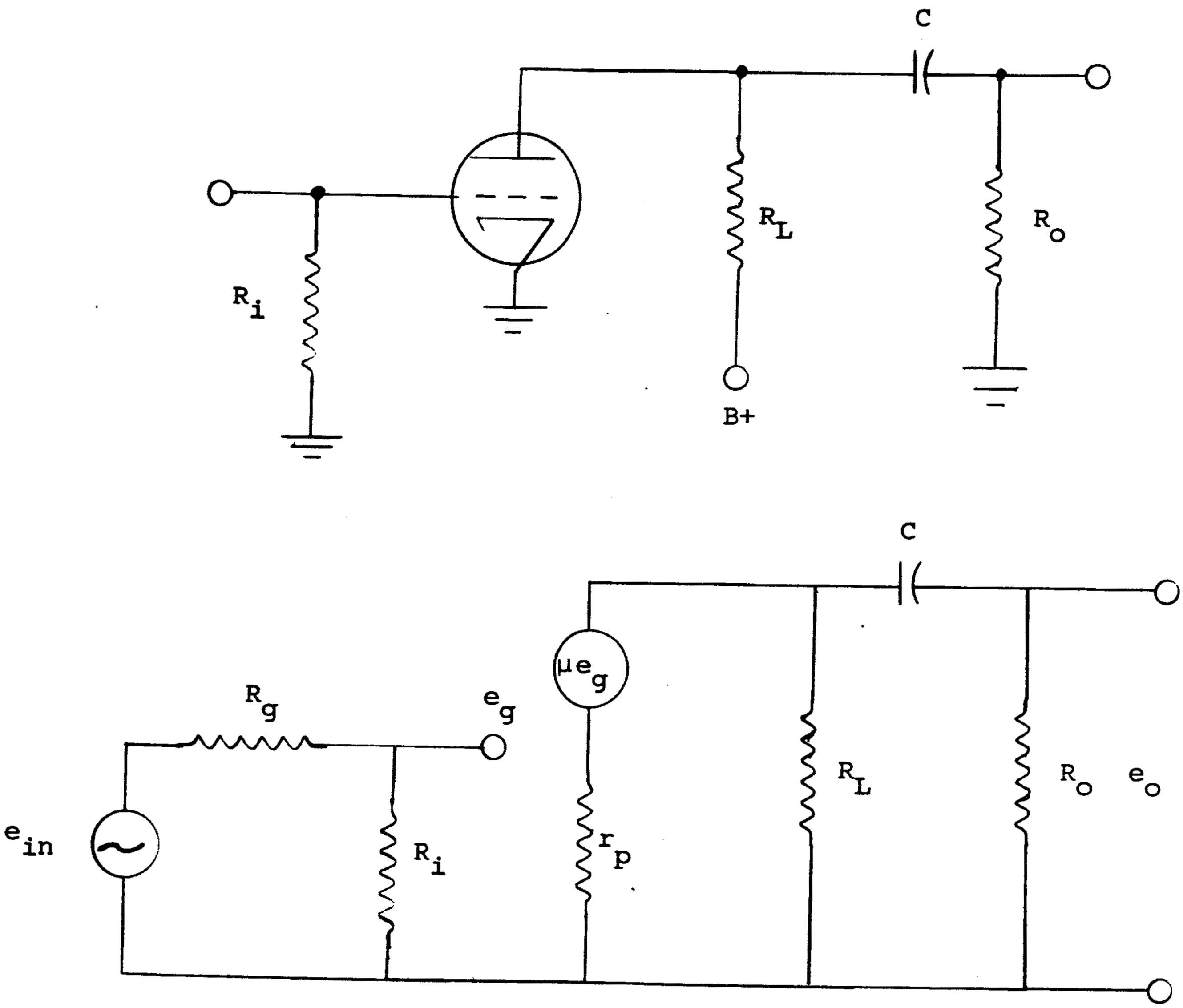


FIGURE 3-1

TRIODE AMPLIFIER AND EQUIVALENT CIRCUIT

where

$$K = \frac{R_i \mu}{(R_g + R_i)}$$

$$d_1 = 1 + r_p/R_L + r_p/R_0$$

$$d_0 = (R_L + r_p)/C\ R_0R_L$$

Clearly this amplifier falls into the general category investigated under the definition of first order transfer functions. The fact that "r_p" is included in both "d_1" and "d_0" complicates the testing required to interrogate the plate resistance. However, "μ" the tube amplification ratio is a linear function in the gain and clearly fits into the parameters already measured. Another note worthy fact in this transfer function is equal order of "S" in both numerator and denominator.

A major assumption in the amplifier transfer function concerns the validity of the model for the tube operation. Consequently, testing signals must be used that satisfy the dynamic range of the model in order to legitimately test the particular transfer function. Let's proceed by obtaining a transfer characteristic for a pentode amplifier as shown in Figure 3-2. The appropriate small signal equivalent circuit in Figure 3-2 is somewhat nonconventional, but for comparison it is desirable to obtain a relation similar to a triode with tube amplification and plate resistance as parameters.

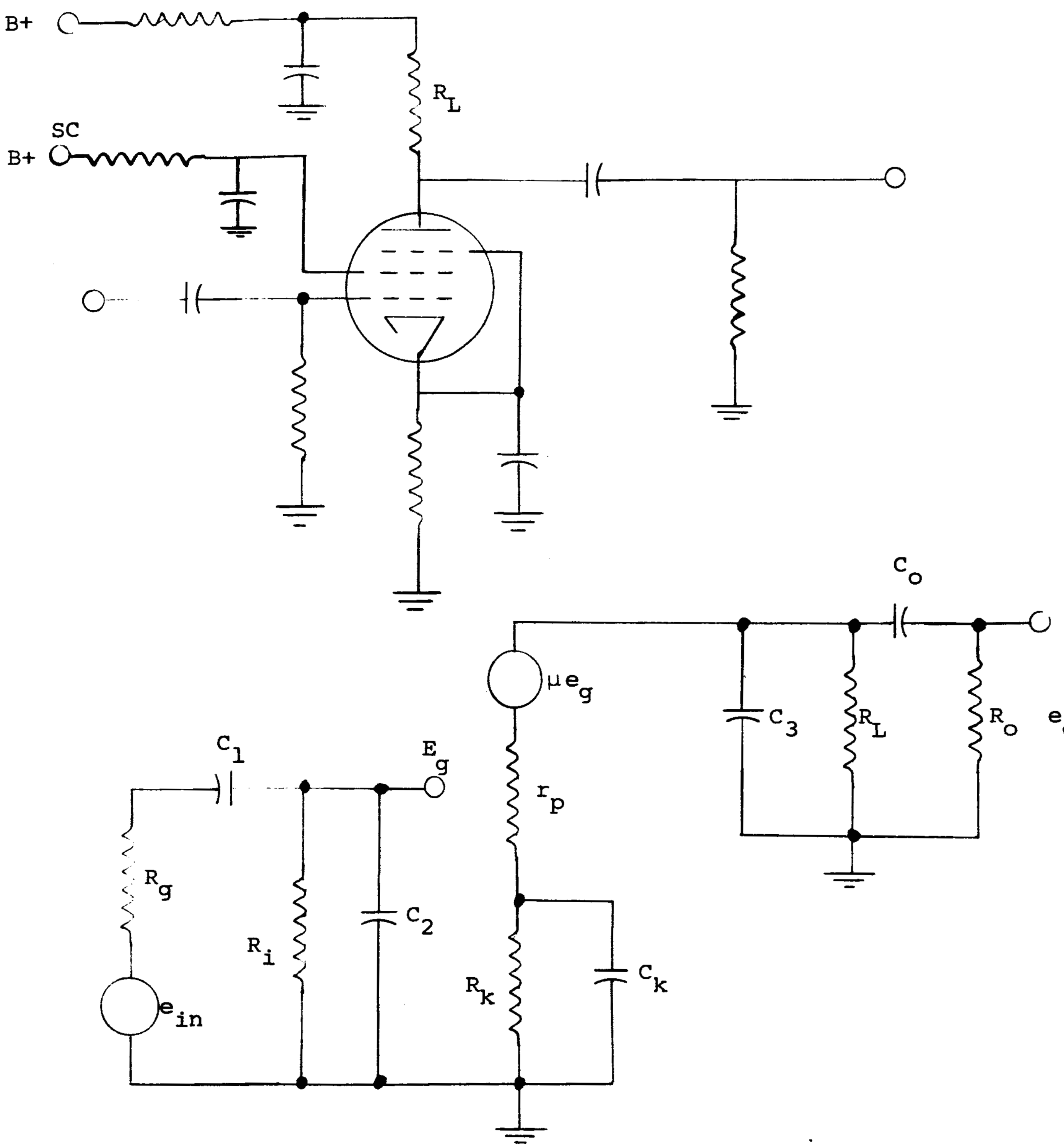


FIGURE 3-2

PENTODE AMPLIFIER AND EQUIVALENT CIRCUIT

The transfer function for the pentode amplifier is derived in Section 3.2.1 as

$$\frac{e_o}{e_{in}} = \frac{\mu R_i C_1 (1 + R_k C_k s)s^2}{(d_5 s^2 + d_4 s + 1)(d_3 s^3 + d_2 s^2 + d_1 s + d_0)}$$

where

$$d_0 = \frac{1}{C_0 R_0} (1 + \frac{R_k + r_p}{R_L})$$

$$d_1 = 1 + \frac{r_p + R_k}{R_0} + \frac{r_p + R_k}{R_L} + \frac{R_k C_k}{R_0 C_0} (1 + \frac{r_p}{R_L})$$

$$d_2 = r_p C_3 + R_k C_k (\frac{C_3}{C_k} + 1 + \frac{r_p}{R_0} + \frac{r_p}{R_L})$$

$$d_3 = R_k C_k r_p C_3$$

$$d_4 = R_g C_1 + R_i C_1 + R_i C_2$$

$$d_5 = R_i R_g C_1 C_2$$

It is interesting to see that plate resistance is contained in every coefficient of the cubic. However, tube transconductance ($G_m = \mu/r_p$) is normally given for a pentode and the possibility exists of examining the active tube parameter as a gain coefficient in this transfer function. Also note that the order of the denominator is high, and it might be difficult to measure each of the circuit parameters because of the laborious calculating required. One other factor involved in this model is signal dynamics, which again just remain within the bounds of the model.

Next, let's examine a transistor amplifier in a similar fashion to the tube circuits in order to allow some comparison of active parameter measurement requirements for transistors. A simple audio amplifier circuit is shown in Figure 5 along with the equivalent circuit in Figure 6. The by-pass around the emitter bias resistor is assumed perfect in order to simplify the algebra.

The transfer function for this transistor amplifier is derived in Section 3.2.1 as

$$\frac{e_0}{e_{in}} = \frac{K\, S^2}{(d_3\, S + 1)\,(d_2\, S^2 + d_1\, S + d_0)}$$

where

$$K = \frac{R_L\, R_0\, C_0\, \alpha_0}{r_e}$$

$$d_0 = \left(\frac{R_{BB} + R_i}{R_{BB}\, R_i}\right) \frac{1}{C_1} + \left(\frac{1 - \alpha_0}{r_e}\right) \frac{1}{C_1}$$

$$d_1 = \frac{C_e}{C_1} + \left(\frac{R_{BB} + R_i}{R_{BB}\, R_i}\right) R_g + \left(\frac{1 - \alpha_0}{r_e}\right) R_g + 1$$

$$d_2 = C_e\, R_g$$

$$d_3 = (R_L + R_0)\, C_0$$

The active device parameters appear in the gain and two coefficients of the quadratic denominator term. Consequently, similar measurements to those required for the pentode amplifier can be applied to this transistor amplifier. In spite of the circuit loading due to the transistor, the transfer characteristic comes out rather simple in terms of the active parameters.

The governing assumption for the development of all three transfer functions is certainly the model validity and the ability to keep signals within model restrictions. A common property clearly contained in all three characteristics is the active parameter in the gain coefficient. In the pentode amplifier and the transistor audio amplifier the third order denominator term and the second order term can be compared if the cubic was factored into a quadratic and first order term. Then both would exhibit second order denominator terms with the active element clearly contained in coefficients.

The relative importance of coefficients in the denominator of a second order transfer function are equivalent to the importance of parameters in an active device. Consequently, a thorough evaluation in any experimental sense of variations in denominator coefficients of a second order system, is required.

Normally, the transistion from one model of an active network to another model is based on properties of the active parameters. For example, a tube equivalent circuit is modified when either dynamic plate resistance or tube gain vary appreciably over the dynamic range of interest. In the mathematical model some of the parameter variations can be included with a non-linear or non-constant function representing the parameter. In a strict mathematical sense, this non-linear effect is difficult to include but experimental observations can be made on the analog computer simulation. If the basic relations obtained for a constant parameter second order system are used in conjunction with a saturation effect type non-linearity, some small range for the model can be established.

3.2.1 Examples of Active Networks

3.2.1.1 Triode Amplifier

The transfer function for the triode amplifier equivalent circuit in Figure 3-1 is easily obtained from circuit equations.

From the input voltage divider

$$e_g = \frac{R_i}{R_g + R_i} e_{in}$$

Labeling currents in the two loop mesh as "I_1" flowing through "r_p" and "I_2" flowing through R_0 we can easily write the matrix relation

$$E = Z\,I$$

or

$$\begin{bmatrix} \mu e_g \\ 0 \end{bmatrix} = \begin{bmatrix} R_L + r_p & -R_L \\ -R_L & \dfrac{C(R_O + R_L) + 1}{SC} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

Where "S" is the Laplace operator.

To obtain the currents we apply cramers rule

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \frac{1}{\Delta} \begin{bmatrix} \frac{\mu eg}{SC}(CR_0 + CR_L + 1) \\ \mu e_g \; R_L \end{bmatrix}$$

and

$$\Delta = \frac{(C\, R_L\, R_0 + C\, r_p\, R_0 + C\, r_p\, R_L)\, S + R_L + r_p}{SC}$$

To obtain the output voltage

$$e_o = I_2\, R_0$$

plus "e_g" into I_2 and obtain

$$e_o = \frac{C\, R_0 R_L\, \mu\, R_i\, e_{in}}{(R_g + R_i)\, [(C\, R_L RO + C\, r_p RO + C\, r_p R_L)\, S + R_L + r_p]}$$

or simplified

$$\frac{e_o}{e_{in}} = \frac{\mu\, R_i\, e_{in}}{(R_g + R_i)\, [(1 + r_p/R_L + r_p/R_0)\, S + \frac{(R_L + r_p)}{C\, R_0\, R_L}]}$$

3.2.1.2 Pentode Amplifier

The transfer function for the pentode amplifier equivalent circuit shown in Figure 3-2 can be obtained in two parts.

First the input filter can be set-up as a voltage divider in terms of impedance as

$$e_g = \frac{Z_2}{Z_1 + Z_2} \quad e_{in}$$

where

$$Z_1 = R_g + \frac{1}{C_1\, S} = \frac{R_g C_1\, S + 1}{C_1\, S}$$

$$Z_2 = \frac{1}{1/R_i + C_2\, S} = \frac{R_i}{1 + R_i C_2\, S}$$

Hence

$$e_g = \frac{R_i\ C_1\ S\ \ \ e_{in}}{(R_i\ R_g\ C_1\ C_2)\ S^2 + (R_g\ C_1 + R_i\ C_1 + R_i\ C_2)\ S + 1}$$

Next, the output filter can be set up as a matrix

$$E = Z\ \ I$$

or

$$\begin{bmatrix} \mu e_g \\ 0 \end{bmatrix} = \begin{bmatrix} r_p + Z_1 + Z_L & -Z_L \\ -Z_L & Z_L + \frac{1}{SC_0} + R_0 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

where

$$Z_1 = \frac{1}{\frac{1}{R_K} + C_K\ S} = \frac{R_K}{1 + R_K\ C_K\ S}$$

$$Z_L = \frac{1}{\frac{1}{R_L} + C_3\ S} = \frac{R_L}{1 + R_L\ C_3\ S}$$

Applying Cramers rule to obtain "I_2"

$$I_2 = \frac{1}{\Delta}\ Z_L\ \mu\ e_g$$

where

$$\Delta = \frac{r_p + Z_1 + Z_L + [C_0 r_p\ Z_L + C_0 r_p\ R_0 + C_0\ Z_1 Z_L + C_0 R_0 (Z_1 + Z_L)] S}{C_0\ S}$$

Since the output is

$$e_o = R_0\ I_2 = \frac{R_0\ Z_L\ \mu\ e_g}{\Delta}$$

which can be expressed as

$$e_o = \frac{\mu\ e_g\ S}{(1 + \frac{r_p + Z_1}{R_0} + \frac{r_p + Z_1}{Z_L})\ S + \frac{r_p + Z_1 + Z_L}{C_0 R_0 Z_L}}$$

With some algebra the impedance can be inserted and the output written as

$$e_o = \frac{\mu \ e_g \ s \ (1 + R_K C_K \ s)}{d_3 \ s^3 + d_2 \ s^2 + d_1 \ s + d_0}$$

where

$$d_3 = R_K C_K r_p C_3$$

$$d_2 = r_p C_3 + R_K C_K \left(\frac{C_3}{C_K} + 1 + r_p/R_0 + r_p/R_L\right)$$

$$d_1 = 1 + \frac{r_p + R_K}{R_0} + \frac{r_p + R_K}{R_L} + \frac{R_K C_K}{C_0 R_0} \ (1 + r_p/R_L)$$

$$d_0 = \frac{1}{C_0 R_0} \ \left(1 + \frac{R_K + r_p}{R_L}\right)$$

Inserting the Grid voltage calculated already

$$e_o = \frac{\mu \ R_i C_1 \ s2 \ (1 + R_K C_K \ s) \ \ e_{in}}{(d_5 s^2 + d_4 \ s+1)(d_3 s^3 + d_2 s^2 + d_1 \ s + d_0)}$$

where

$$d_5 = R_i R_g C_1 C_2$$

$$d_4 = R_g C_1 + R_i C_1 + R_i C_2$$

Hence the pentode amplifier transfer ratio is

$$\frac{e_o}{e_{in}} = \frac{\mu \ R_i \ C_1 \ (1 + R_K \ C_K \ s) \ s^2}{d_5 \ s^2 + d_4 \ s + 1)(d_3 \ s^3 + d_2 \ s^2 + d_1 \ s + d_0)}$$

3.2.1.3 Transistor Amplifier

The transfer function for the equivalent circuit shown in Figure 3-3 can be obtained in two parts. In this case, the

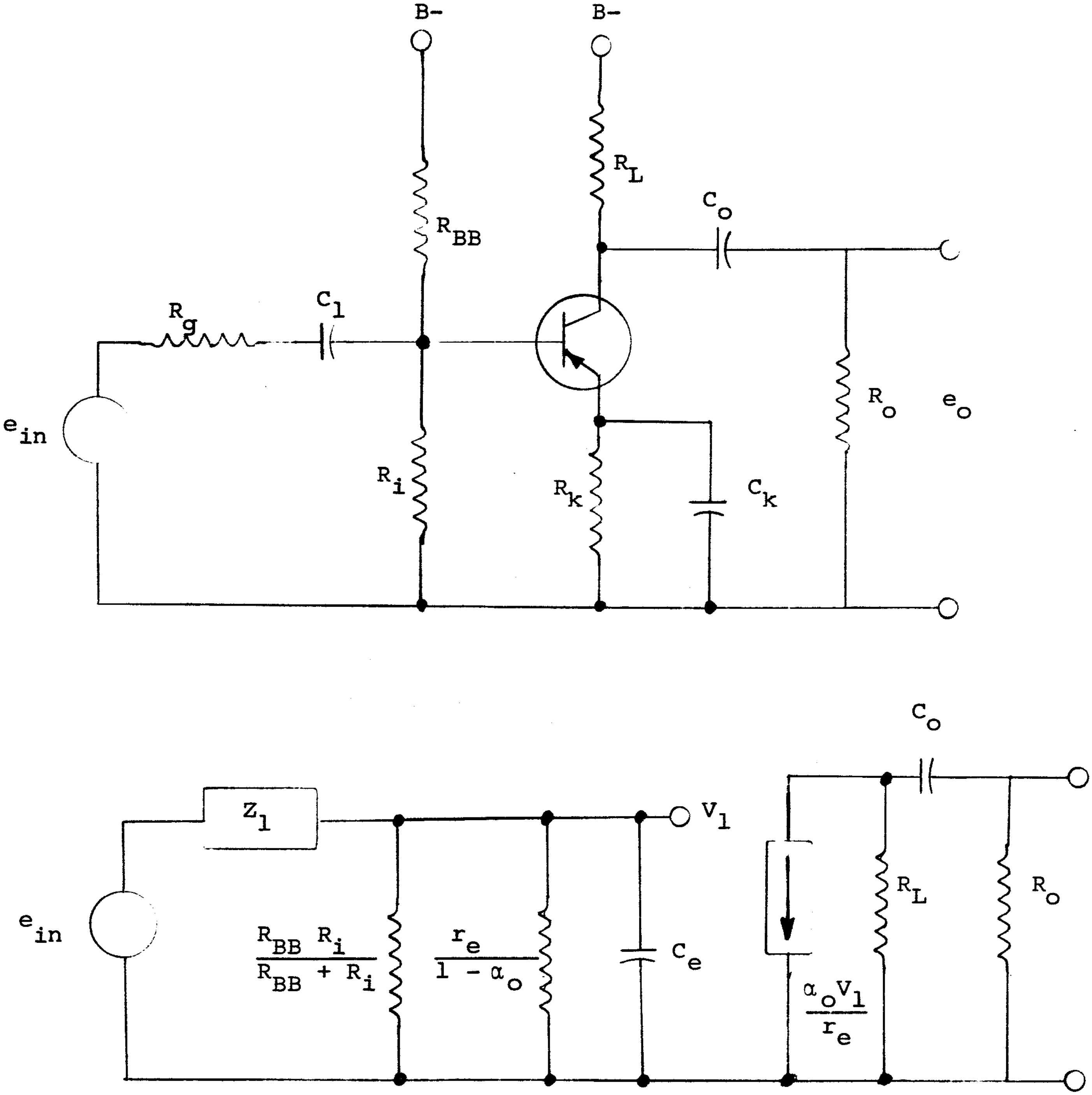


FIGURE 3-3

TRANSISTOR AUDIO AMPLIFIER AND EQUIVALENT CIRCUIT first part is merely a voltage divider and the second part a current divider.

First, the input filter can be set-up as

$$V_1 = \frac{Z_2}{Z_1 + Z_2} \quad e_{in}$$

where

$$Z_1 = R_g + \frac{1}{C_1 S} = \frac{R_g C_1 S + 1}{C_1 S}$$

$$Z_2 = \frac{1}{\frac{R_{BB} + R_i}{R_{BB} R_i} + \frac{1 - \alpha_0}{r_e} + C_e S}$$

or

$$Z_2 = \frac{r_e R_{BB} R_i}{(R_{BB} + R_i) r_e + (1 - \alpha_0) R_{BB} R_i + r_e R_{BB} R_i C_e S}$$

Hence, after simplifying

$$V_1 = \frac{S \quad e_{in}}{d_2 S^2 + d_1 S + d_0}$$

where

$$d_2 = C_e R_g$$

$$d_1 = \frac{C_e}{C_1} + \frac{(R_{BB} + R_i)}{R_{BB} R_i} R_g + (1 - \alpha_0) \frac{R_g}{r_e} + 1$$

$$d_o = \frac{(R_{BB} + R_i)}{R_{BB} R_i C_1} + \frac{(1 - \alpha_0}{r_e C_1}$$

Next the output current divider is

$$I_0 = \frac{R_L}{R_L + Z_0} \quad (\frac{\alpha_0 V_1}{r_e})$$

where

$$e_o = I_0 R_0 = \frac{R_L R_0 C_0 S (\alpha_0 V_1)}{r_e (R_L + R_0) C_0 S + 1)}$$

or in a simplified form

$$\frac{e_o}{e_{in}} = \frac{K \ s^2}{(d_3 \ s + 1)(d_2 \ s^2 + d_1 \ s + d_0)}$$

where

$$K = \frac{R_L R_0 C_0 \alpha_0}{r_e}$$

$$d_3 = (R_L + R_0) \ C_0$$

3.2.2 Initial Condition in a System

Initial conditions and constant source values, such as a D-C bias, both enter a transfer function in a very similar way as many text books prove. The question here is one of measureability of this source value and where it appears in the transfer characteristic. To observe these properties, consider the definition of a Laplace transform.

$$H(S) = \int_0^{\infty} f(t) \exp(-st) \, dt$$

where s = Laplace transform operator

$f(t)$ = Time function of the source

Normally an initial value theorem is stated so that the initial value depends upon a limiting process where "S" approaches infinity. Then this transfer function can only take on non-vanishing values if the exponential term does not vanish. This can occur only for very small values of t, and the result in the limit is the initial condition.

Consider a typical source in the general form

$$f(t) = M\,t^n$$

where

$$n = 0, 1, 2, \ldots\ a$$

$$M = \text{Scale factor}$$

clearly

$$H(S) = \frac{M\,n!}{S^{n+1}}$$

If f(t) is merely a bias contained in a voltage equation, n = 0 and

$$\lim_{S \to \infty} H(S) = \lim_{S \to \infty} \frac{M}{S} = 0$$

Since S contains a real and imaginary part ($S = \alpha + j\,w$) this clearly shows that the real part of S is required to go to infinity as well as the imaginary part. This is required because the integral of an exponential along only a vertical line would become an undefined quantity (except in a special impulse sense).

This would not be clear if the theorem is written as follows:

$$\lim_{S \to \infty} S^{n+1} F(S) = \left[\frac{d^n f(t)}{dt^n}\right] = f^{(n)}(0)$$

From this discussion it is apparent that initial conditions appear normally as poles at the origin.

The next concern is the measureability of these poles. Since the growing exponential technique is based on a transient responce. The non-transient, D-C or steady values can not be measured by applying this technique. However, the opposite question concerning the use of growing exponentials in testing a system with a pole at zero does arise. The normal approach in testing a system with a D-C value is too block the D-C out of the measuring circuit. This can be accomplished in an electrical network with a capacitor in series with the output to the measuring equipment. A typical transfer characteristic for this blocking action is

$$H_B(S) = \frac{C\ R_0\ S}{C\ R_0\ S+1} = \frac{S}{S + \frac{1}{C\ R_0}}$$

Obviously the pole at the origin is removed and replaced with a pole at

$$P = \frac{1}{C\ R_0}$$

This complicates the mathematics in obtaining an estimator for the growing exponential technique in that it adds a pole to be considered in evaluation of residues. However, the basic mathematics is in no way limited since the presence of additional poles is not presently considered a limiting factor.

One requirement inferred in a system in this discussion is stability. In order to test with growing exponential signals, a basic requirement is that bounded or controlled input signals produce controlled or bounded output signals. With this in mind, problems associated with stability and initial conditions must be assumed answered by another means.

3.3 REDUCING TRANSFER FUNCTIONS

All of the techniques study thus far depend upon a knowledge of the system transfer function. This may be difficult to obtain for complex physical systems, however, at least three general methods are available. The first method is where the system may obey a physical model approximation very accurately within some specified dynamic range. In this instance, calculations based on the model will result in a transfer function in which parameters will be calculated or experimentally determined. The second method is a Bode frequency plot of magnitude and phase for both input and output may be obtained with an appropriate variable frequency sinusoidal generator. This will lead to an approximation of the transfer function if appropriate phase and amplitude characteristics are properly analyzed. The third method is a transient test where a step or ramp transient input may be used and the transfer function again approximated in the time domain.

The approximation of an actual function must always contain some error. Control theory usually demands a simple transfer function, thus, the error is dictated by relative importance of particular poles and zeros. In the case of a large number of poles and zeros, the dynamic frequency range of the model will be restricted. Thus, simplicity and/or reduction of poles and zeros must be considered to emphasize specific dominant poles and zeros.

One way of evaluating important system characteristics is to split the system transfer function up into its important parts. The general area of control system compensation has applied this concept in order to manipulate poles and zeros around the complex plane with the objective of achieving a particular time response. This same scheme is applicable in measuring system performance and it seems worthwhile at this point to investigate some of the results.

A simple demonstration of splitting off important system transfer function parts is easily seen if a difference scheme is considered. For example, let the complete transfer function consist of

$$G_T(S) = G_1(S) \quad G_2(S)$$

Suppose the important factors are included in $G_2(S)$.

Then $\quad G_T(S) - G_3(S) = G_2(S)$

and clearly

$$G_3(S) = G_2(S) \; [G_1(S) - 1]$$

The result indicates that an effective system transfer function $G_2(S)$ can be obtained by subtracting a more complex function away from the total system function. This result seems trivial, but compare this with the ordinary methods used when the system characteristic is complicated. Pole and zero locations are observed in the calculated transfer function and thrown out when appropriate, to simplify the transfer function with little regard for the actual dynamic range. The measureability of parameters included in the modified transfer function will not be the same for these two cases.

Another means of reducing a complicated function is by cascade or feedback compensation. This introduces the possibility of poles and zeros entering the measurable transfer function because of imperfect cancellation. However, it may offer the possibility of a simplified function for reducing the system compared to the simple difference scheme above.

The reduction process offers the possibility of investigating higher order systems in terms of important parameters in lower order systems. However, limitations will exist if the linearity of a system function is in question. The position of the transfer function in the algebra used, requires a commutative property exhibited only for linear time invariant systems.

3.4 LAGUERRE FUNCTION SET

The growing exponential signals are matched to partial derivatives with respect to particular parameters. Consequently, some approximation of the partial derivative system is required. This approximation problem can be difficult and it seems worthwhile to examine the properties of the functions used. For example, when many parameters are related to the same characteristic frequency, the approximating function can involve the Laguerre set of orthonormal functions. This set has all of the properties of a finite dimensional coordinate system. That is, any particular function can be expressed in terms of a sum of a finite number of terms. The form of each term is a constant multiplied by one of the members of the Laguerre set. Since each member of the Laguerre set is orthogonal to all others over the interval t = 0 to infinity, and the sum contains only "N" terms, the set clearly exhibits all of the normal properties of a finite dimensional coordinate system. Two properties that this Laguerre set exhibits are worth noting. First each member is contained in all higher order members, for example L_2 contains L_1 and L_0. The second property is the exponential multiplier times a specific polynomial in the time parameter. This is exactly suited to approximating linear system impulse response functions for causal systems.

The following six functions represent the first six Laguerre functions.

$$L_0 = \sqrt{2p} \quad \exp^{(-pt)}$$

$$L_1 = \sqrt{2p}\ (2pt - 1) \quad \exp^{(-pt)}$$

$$L_2 = \sqrt{2p}\ (2p^2t^2 - 4pt + 1) \quad \exp^{(-pt)}$$

$$L_3 = \sqrt{2p}\ (4/3\ p^3t^3 - 6p^2t^2 + 6pt - 1) \quad \exp^{(-pt)}$$

$$L_4 = \sqrt{2p}\ (2/3\ p^4t^4 - 16/3\ p^3t^3 + 12p^2t^2 - 8pt + 1) \quad \exp^{(-pt)}$$

$$L_5 = \sqrt{2p}\ (4/15\ p^5t^5 - 10/3\ p^4t^4 + 40/3\ p^3t^3 - 20p^2t^2$$

$$+ 10pt - 1) \quad \exp^{(-pt)}$$

and the general term is

$$L_n = \sqrt{2p}\ \left[\frac{(2pt)^n}{n!} - \frac{n(2pt)^{n-1}}{(n-1)!} + \frac{n(n-1)\ (2pt)^{n-2}}{2!\ (n-2)!} \right.$$

$$\left. - \frac{n(n-1)\ (n-2)\ (2pt)^{n-3}}{3!\ (n-3)!} + \cdot \cdot \cdot + (-1)^n \right] \exp^{(-pt)}$$

where

p = the pole location or one over the time constant

t = time

n = 0, 1, 2, 3, - - - -, N

The six functions are calculated for a pole location of one half or a time constant of two seconds and are shown in Figure 3-4. A short digital program was written so that values of all

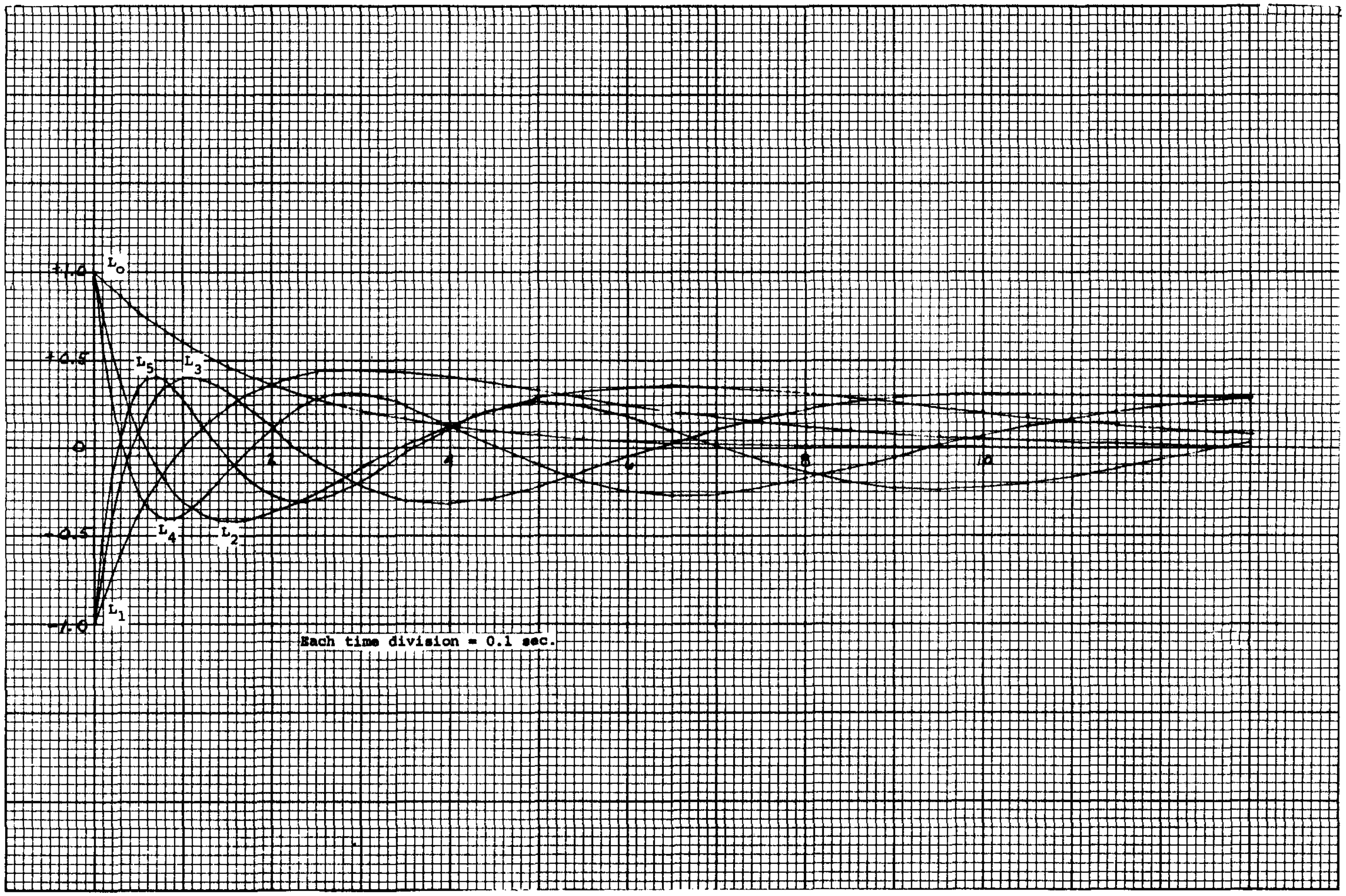


FIGURE 3-4 LAGUERRE FUNCTIONS L_0 TO L_5 WITH A 2 SECOND TIME CONSTANT

six of these functions could easily be obtained for use as initial conditions in the specialized growing exponential generator. All that is required for program use is the particular pole location. Any specified time can be evaluated or the points representing the entire function when particularly low values are desired.

3.5 SECOND-ORDER SYSTEM - DENOMINATOR PARAMETERS

The transfer function is the same as that on page 41 of the Phase A report, namely,

$$H(S) = \frac{C_1S + C_0}{S^2 + d_1S + d_0} = \frac{S + 5}{S^2 + 2S + 10},$$

which is also written

$$H(S) = \frac{C_1S + C_0}{(S + \alpha)^2 + \beta^2}$$

The partial derivatives of H(S), giving the component partial systems corresponding to d_0 and d_1, are

$$\frac{\partial H}{\partial d_1} = \frac{-S}{S^2 + d_1S + d_0} \quad H(S) = \frac{-S(C_1S + C_0}{\left[(S + \alpha)^2 + \beta^2\right]^2}$$

$$\frac{\partial H}{\partial d_1} = \frac{-1}{S^2 + d_1S + d_0} \quad H(S) = \frac{-(C_1S + C_0)}{\left[(S + \alpha)^2 + \beta^2\right]^2}$$

The probing signals are the same as on page 42 of the Phase A report, namely,

$$\Phi_1(s) = \sqrt{2\alpha}\ \frac{(-s + \sqrt{\alpha^2 + \beta^2})}{(-s + \alpha)^2 + \beta^2}$$

and

$$\Phi_2 = \sqrt{2\alpha}\ \frac{(-s - \sqrt{\alpha^2 + \beta^2})\left[(-s - \alpha)^2 + \beta^2\right]}{\left[(-s + \alpha)^2 + \beta^2\right]^2}$$

The matrix components for the partial systems are found by integrating as in the Phase A report, but the work involved is much greater because of the greater complexity of the integrands. As an example, the matrix component $(h_{11})_{d_1}$ is found from

$$(h_{11})_{d_1} = \frac{1}{2\pi j}\int_c \left(\frac{\sqrt{2\alpha}\ (s + \sqrt{\alpha^2 + \beta^2})}{\left[(s + \alpha)^2 + \beta^2\right]}\right)\left(\frac{-s(c_1 s + c_0)}{\left[(s + \alpha)^2 + \beta^2\right]^2}\right)$$

$$\left(\frac{\sqrt{2\alpha}\ (-s + \sqrt{\alpha^2 + \beta^2})}{\left[(-s + \alpha)^2 + \beta^2\right]}\right) ds$$

$$= \frac{2\alpha}{2\pi j}\int_c \frac{(s^2 - \alpha^2 - \beta^2)\ (c_1 s + c_0)\ (s)\ ds}{\left[(s + \alpha)^2 + \beta^2\right]^3\left[(s - \alpha)^2 + \beta^2\right]}$$

Evaluation of this integral is extremely tedious and affords numerous opportunities for making errors. This necessitates a long procedure of solution and correction cycles in order to obtain a general solution. The final results are

$$H_{d1} = \begin{bmatrix} -\frac{1}{16\,\alpha^2}\left(c_1 + \frac{c_o\alpha}{\alpha^2+\beta^2}\right), & \left[\frac{1}{32\alpha^2(\alpha^2+\beta^2)} \left(c_1\alpha^2 \frac{2\alpha\sqrt{\alpha^2+\beta^2}}{\alpha^2+\beta^2} - c_1\alpha^2 - \frac{c_o\,\beta^2}{\sqrt{\alpha^2+\beta^2}}\right)\right] \\ 0, & \left(-\frac{1}{16\,\alpha^2}\left(c_1 + \frac{c_o\,\alpha}{\alpha^2+\beta^2}\right)\right) \end{bmatrix}$$

and

$$H_{do} = \begin{bmatrix} \frac{-1}{4\alpha^2(\alpha^2+\beta^2)}\left[c_1\alpha + c_o\frac{\alpha^2-\beta^2}{\alpha^2+\beta^2}\right], & \frac{-1}{8\alpha^2(\alpha^2+\beta^2)}\left[c_1\sqrt{\alpha^2+\beta^2} - \frac{2\,c_o\alpha^2}{\alpha^2+\beta^2}\right] \\ 0, & \frac{-1}{4\alpha^2(\alpha^2+\beta^2)}\left[c_1\alpha + c_o\left(\frac{\alpha^2-\beta^2}{\alpha^2+\beta^2}\right)\right] \end{bmatrix}.$$

The coefficients of $\Phi_1(s)$ and $\Phi_2(s)$ are chosen to be cos ψ and sin ψ from constant energy consideration as in the Phase A report and the parameter modulation matrix is formed as

$$M = \begin{bmatrix} M_{d1} & M_{do} \end{bmatrix}$$

where $M_{d1} = [H_{d1}] \begin{bmatrix} \cos\psi \\ \sin\psi \end{bmatrix}$

and $M_{do} = [H_{do}] \begin{bmatrix} \cos\psi \\ \sin\psi \end{bmatrix}$

The numerical results for $d_1 = 2$ and $d_o = 10$, which make $\alpha = 1$ and $\beta = 3$, are

$$M = \begin{bmatrix} -0.009124 & -0.01875 \\ -0.005405 & 0.015 \end{bmatrix}$$

and

$$M^{-1} = \begin{bmatrix} -62.97 & -78.71 \\ -22.69 & 38.30 \end{bmatrix}$$

which is normalized to

$$\text{normalized } M^{-1} = \begin{bmatrix} 1.644 & 2.055 \\ 0.5924 & -1 \end{bmatrix}$$

The testing is carried out with an analog computer, using the same arrangement as when testing the numerator parameters of the second order system (see the Phase A report) except that the estimator is changed to reflect the new M^{-1}. See Figure 3-5. The probing signal is shown in Figure 3-6.

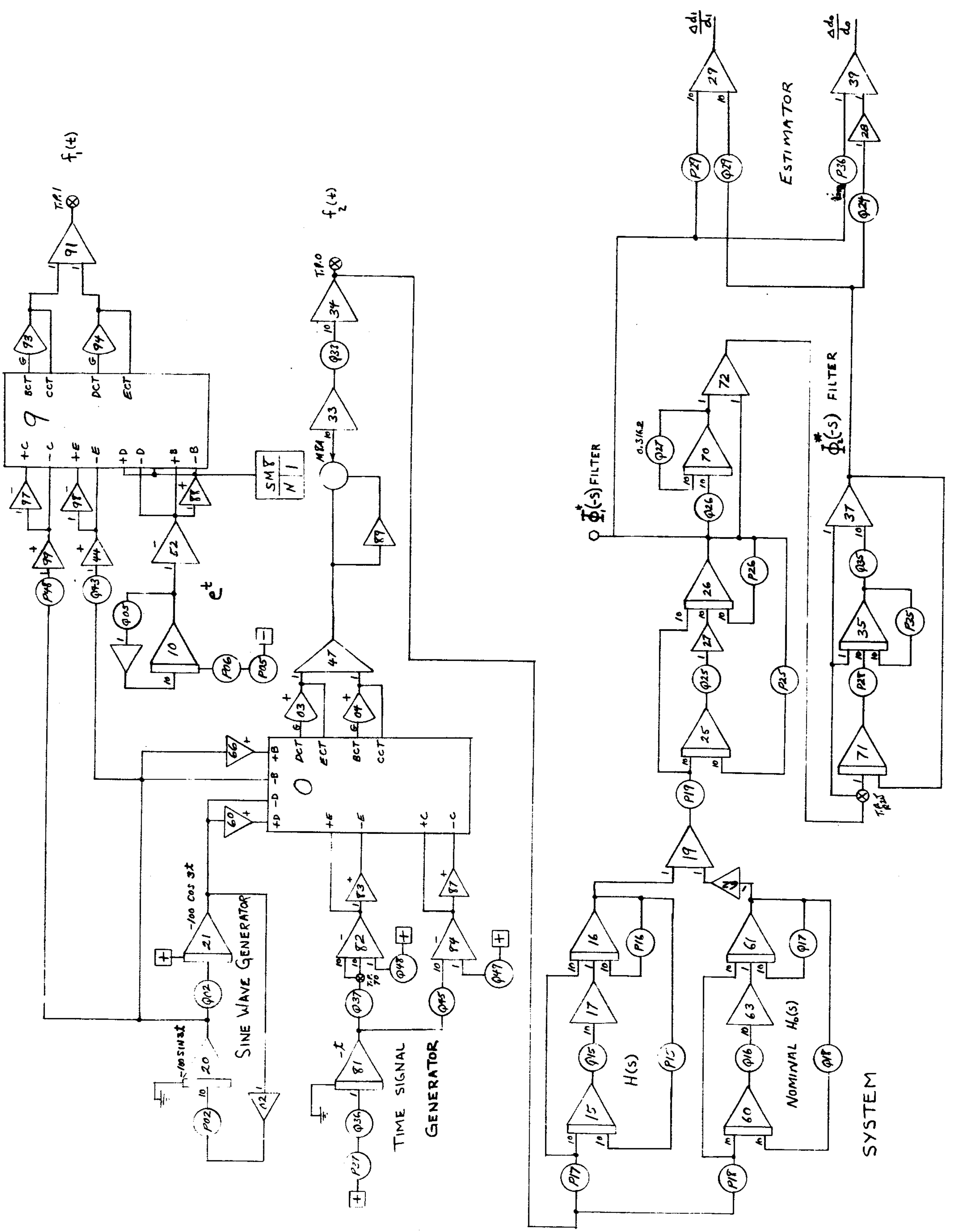


FIGURE 3-5
SECOND ORDER SYSTEM
MEASURING SET-UP

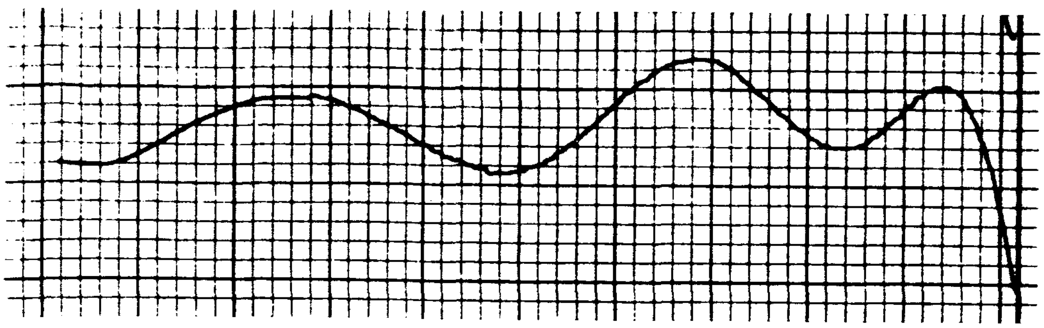


Figure 3-6

Input Probing Signal

3.5.1 EXPERIMENTATION

The system was tested first with variations of d_1 and d_o of 2, 4, 6, 8, 10, 20 and 30% under otherwise nominal conditions. Variations in d_o were readily measured up to 10% from nominal, but there was considerable influence by d_o variations on the estimator channel which should measure d_1 variations, so that the latter could not be measured by themselves. However, by plotting d_1 versus d_o on a set of contours of constant d_1 and d_o variations of both parameters can be measured. Alternatively, when limits have been set on d_o and d_1, contours such as those in Figure 3-12 can be used to decide whether or not the system is acceptable.

Although linearity of measurements became poorer as the variations approached 30%, as seen in Figure 3-8 and 3-10. Nevertheless, Figure 3-14 indicates that the linearity was remarkably good up to at least ten percent. Deteriorating linearity can probably be attributed to the inaccuracy incurred by not using the higher order terms in the Taylor

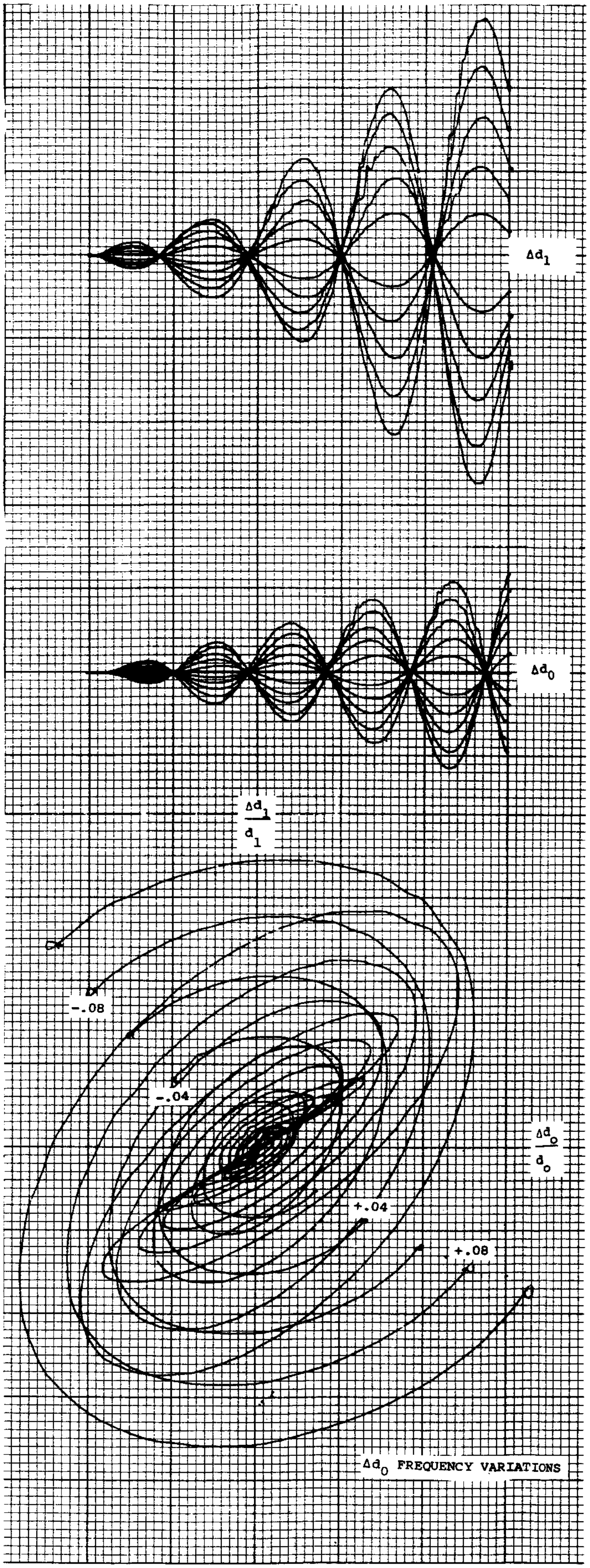


FIGURE 3-7 Δd_0 FREQUENCY VARIATIONS WITH NO VARIATION IN d_1 (SMALL VARIATIONS)

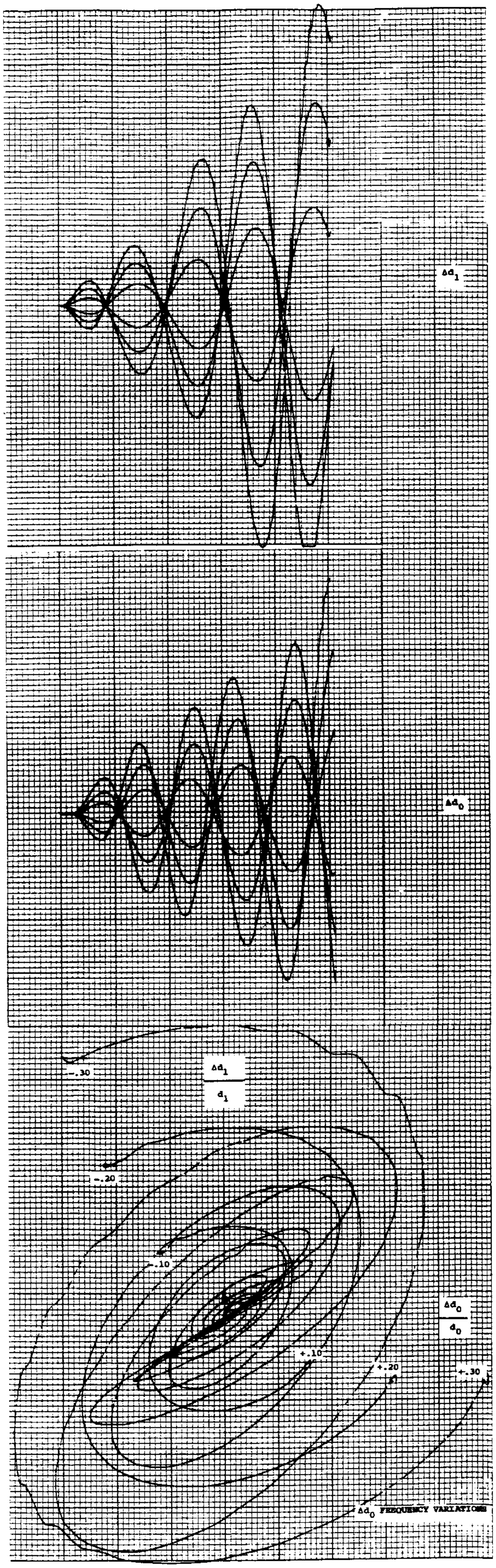


FIGURE 3-8 Δd_0 FREQUENCY VARIATIONS WITH NO VARIATION IN d_1 (LARGE VARIATIONS)

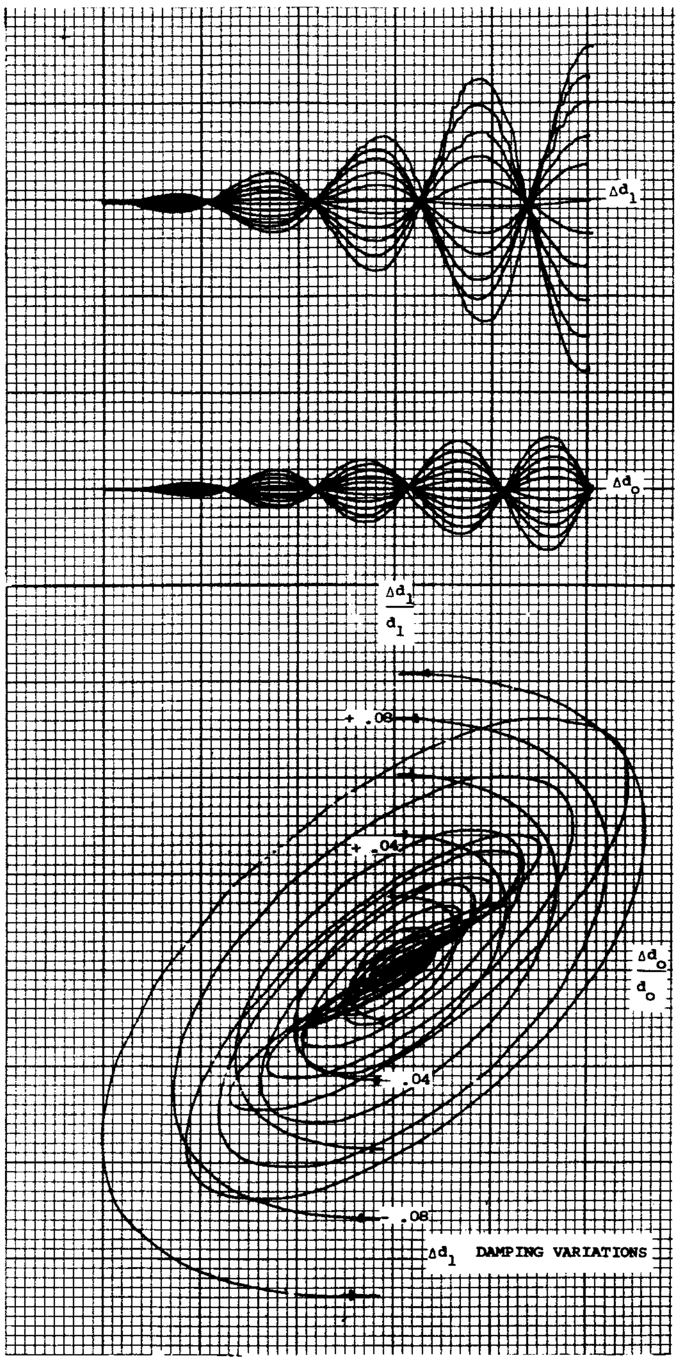


FIGURE 3-9 Δd_1 DAMPING VARIATIONS WITH NO VARIATTONS IN d_0 (SMALL VARIATION)

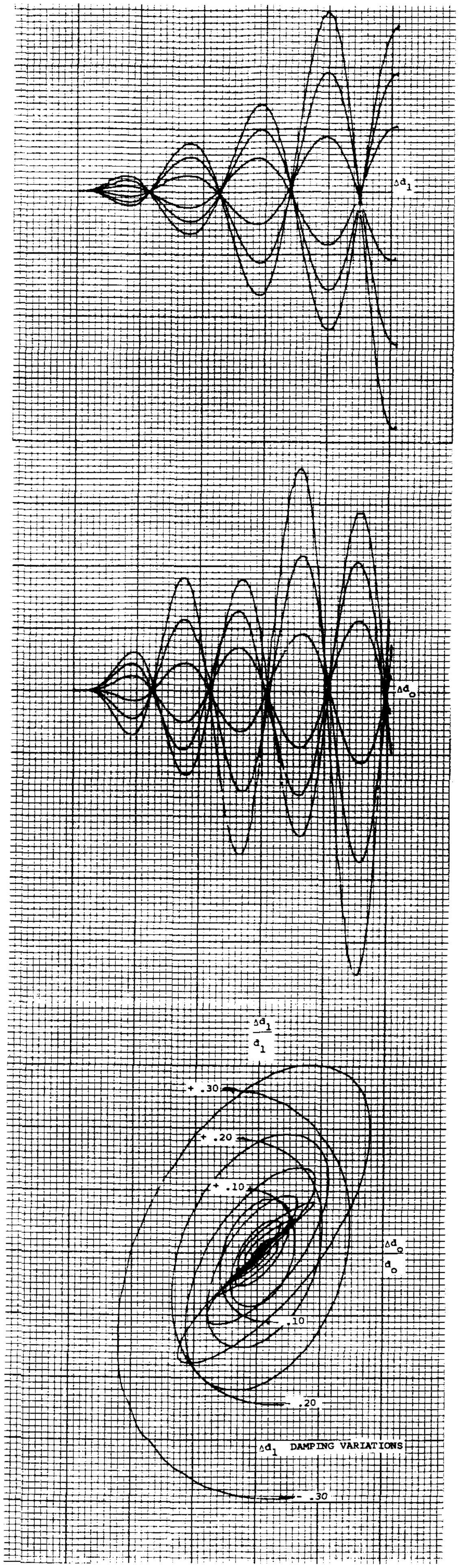


FIGURE 3-10 Δd_1 DAMPING VARIATIONS WITH NO VARIATIONS IN d_0 (LARGE VARIATIONS)

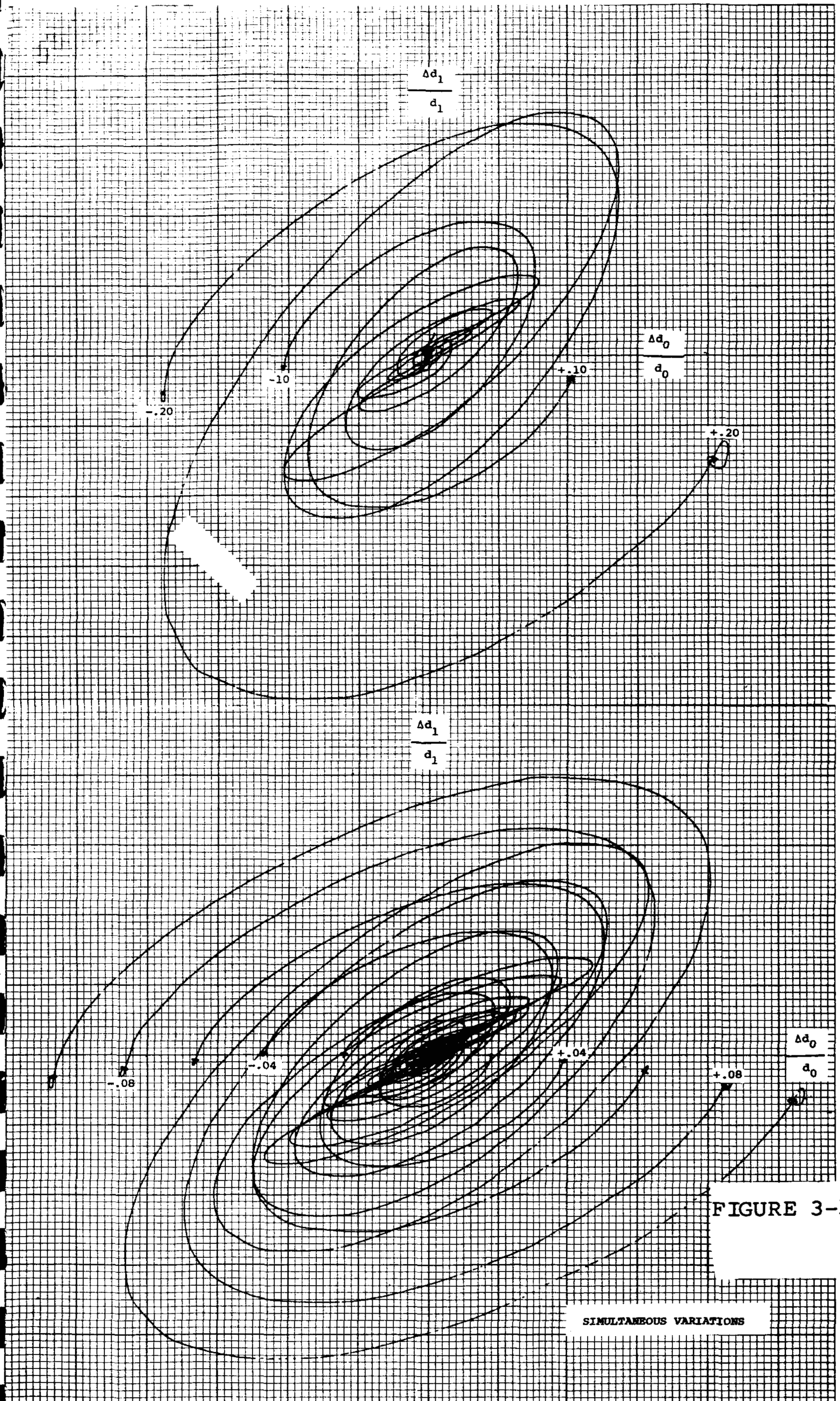


FIGURE 3-11 SIMULTANEOUS FREQUENCY AND DAMPING VARIATIONS

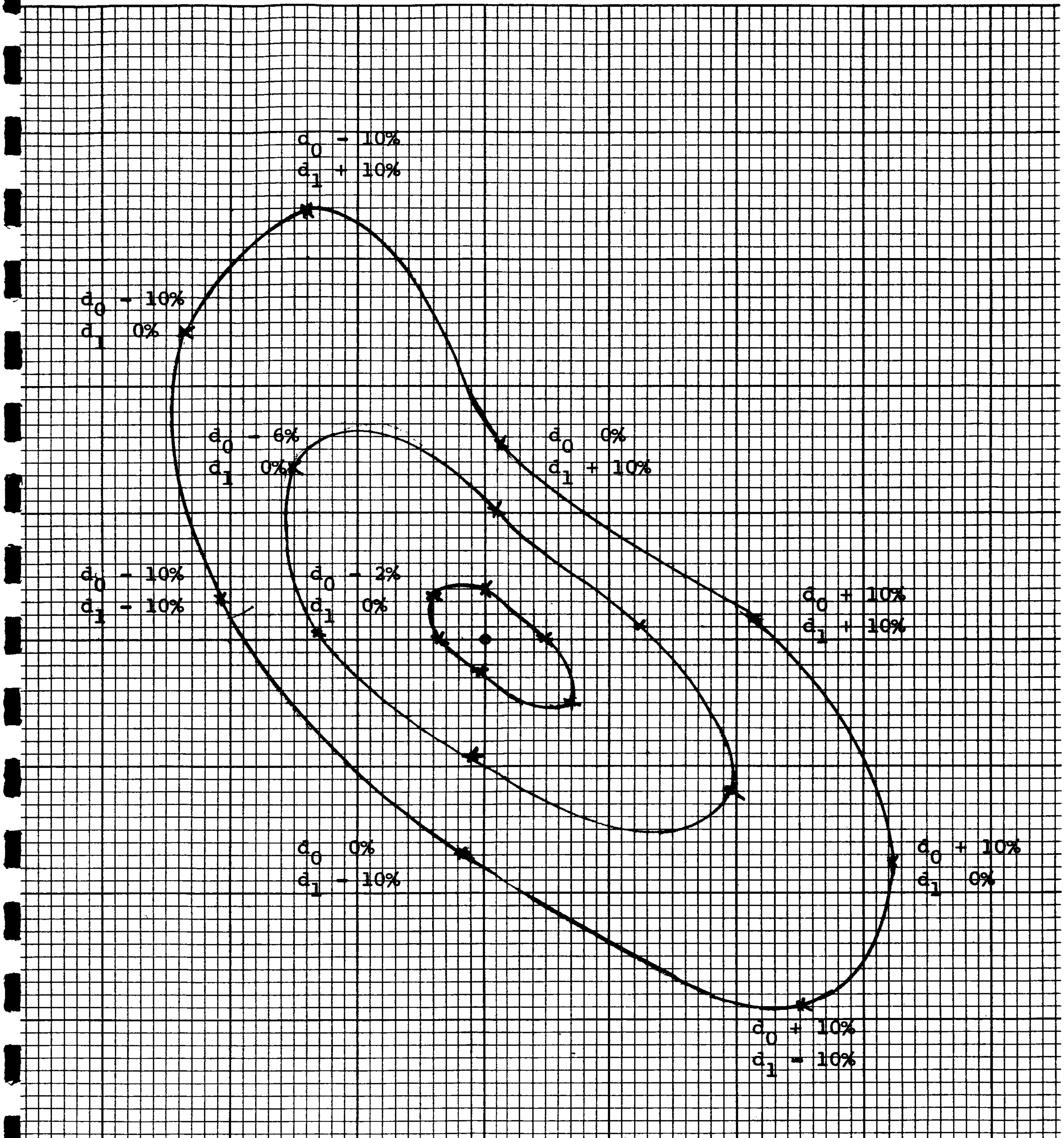


FIGURE 3-12. CONTOURS RELATING LIMITS OF PARAMETERS TO GO-NO-GO TESTING

Series expansion into partial systems (reference Phase A report, page 12).

It was found that the isolation of d_o measurements could be optimized by slight adjustment of the estimator, so that the estimator used in the tests became

$$M^{-1} = \begin{matrix} 1.644 & 2.055 \\ 0.7954 & -1 \end{matrix}$$

3.5.2 NON-LINEAR DAMPING

The gain of the feedback element controlling damping (P16 in Figure 3-5) was varied non-linearly as in Figure 3-13. Measurements were made with a linear range of 18 and 36 volts. The results are shown in Figures 3-15 and 3-16. Plots comparing the measurements of parameter variations are shown in Figure 3-14. The effect of the damping non-linearity was to offset the measurements by an approximate value without appreciably affecting the slopes of the plots of estimated versus actual variations of the parameters.

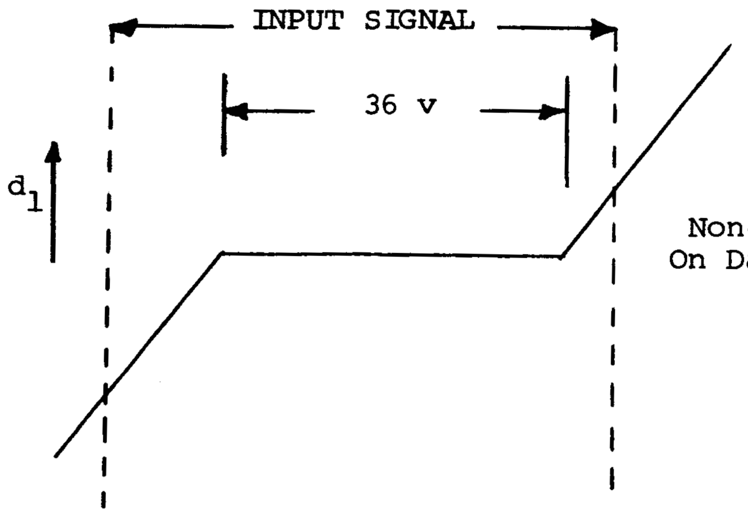


Figure 3-13

Non-Linear Deadband On Damping Coefficient

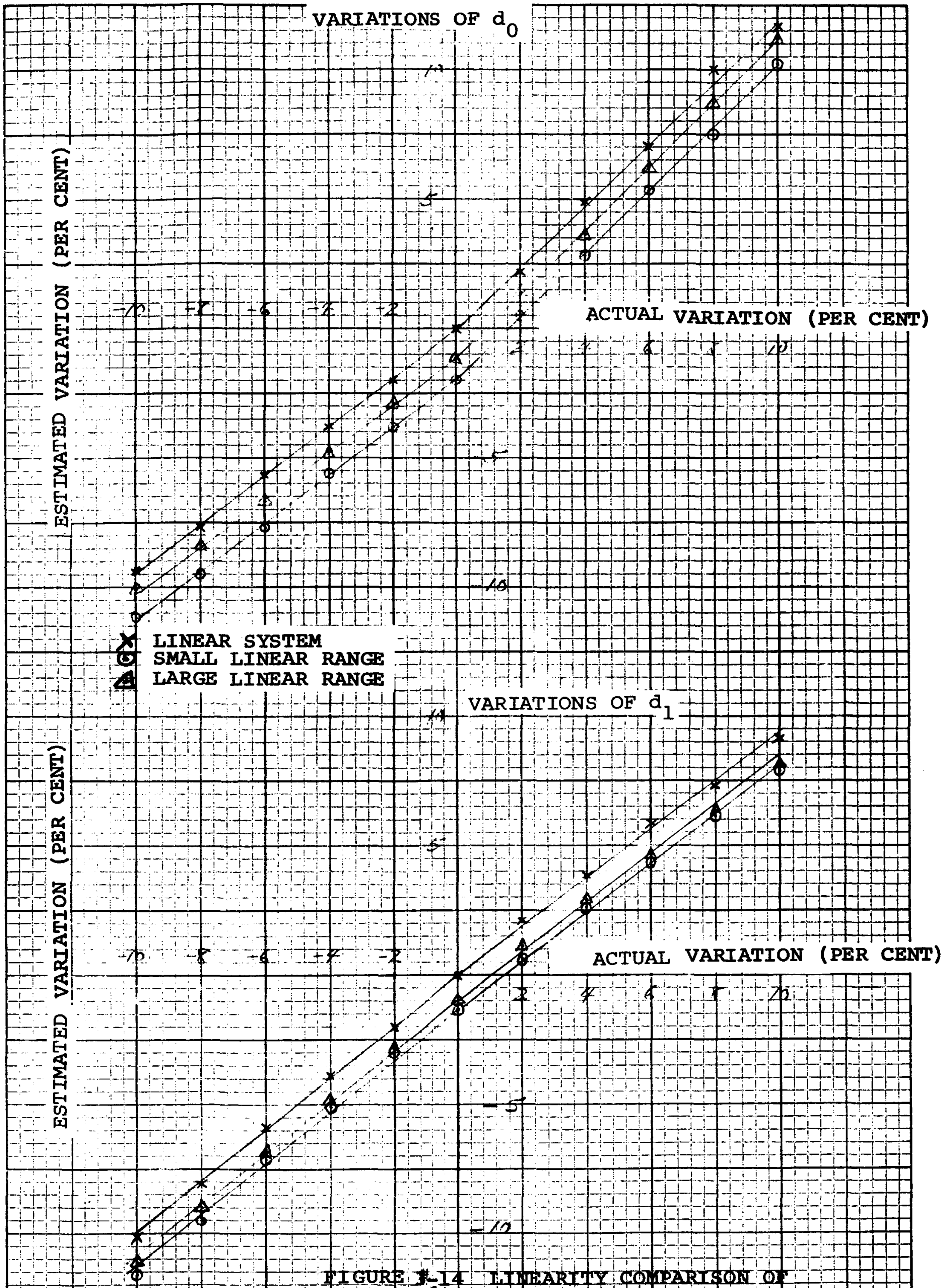


FIGURE 3-14 LINEARITY COMPARISON OF ESTIMATED PARAMETER VARIATIONS

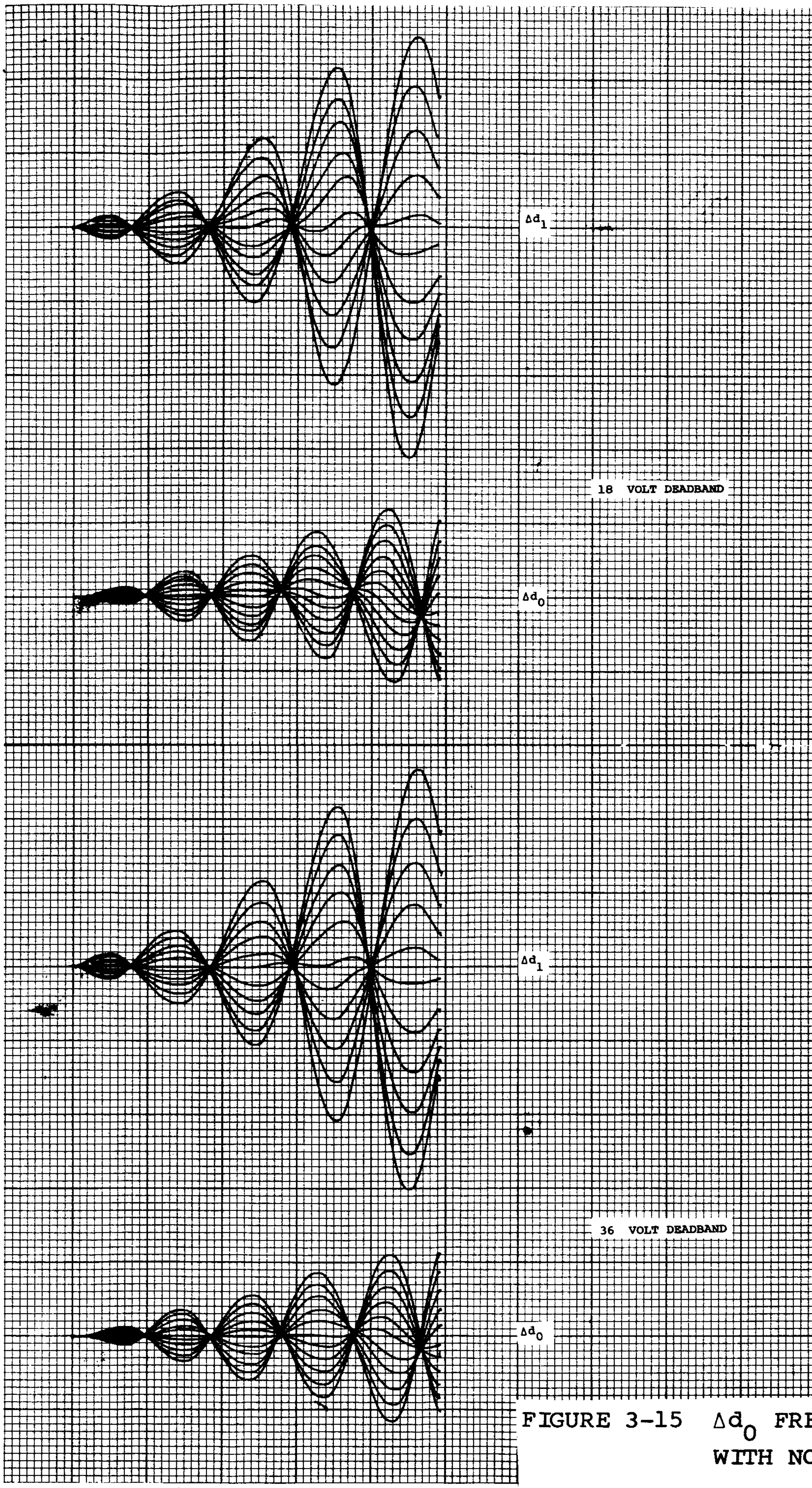


FIGURE 3-15 Δd_0 FREQUENCY VARIATIONS WITH NON-LINEAR DAMPING

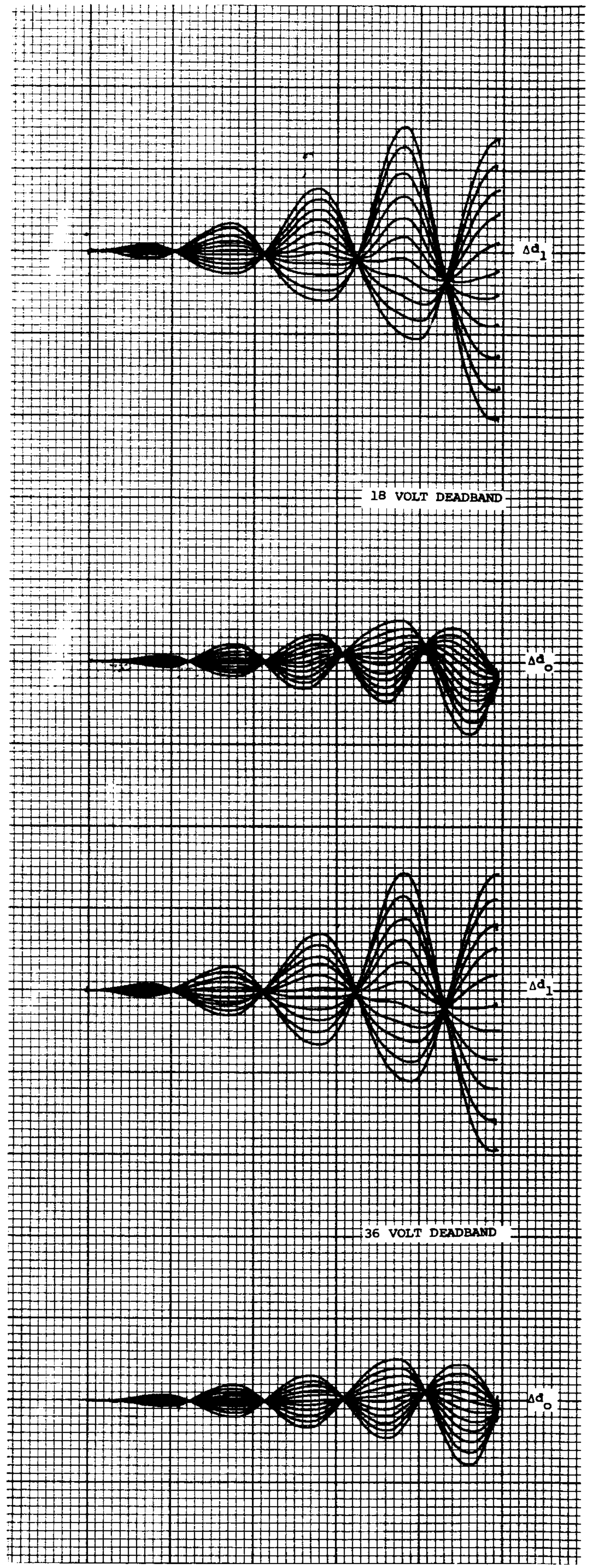


FIGURE 3-16 Δd_1 DAMPING VARIATIONS WITH NON-LINEAR DAMPING (MAINTAINING CONSTANT DEADBAND LENGTH)

The isolation of the measurement of d_o variations from d_1 variations was somewhat deteriorated by inclusion of the non-linearity, as can be seen in Figure 3-16, where there is some variation of the d_o measurement from an initial offset value. (As mentioned earlier the measurement of d_1 could not be isolated from d_o variations, even with a linear system).

3.5.3 NOISE TESTS

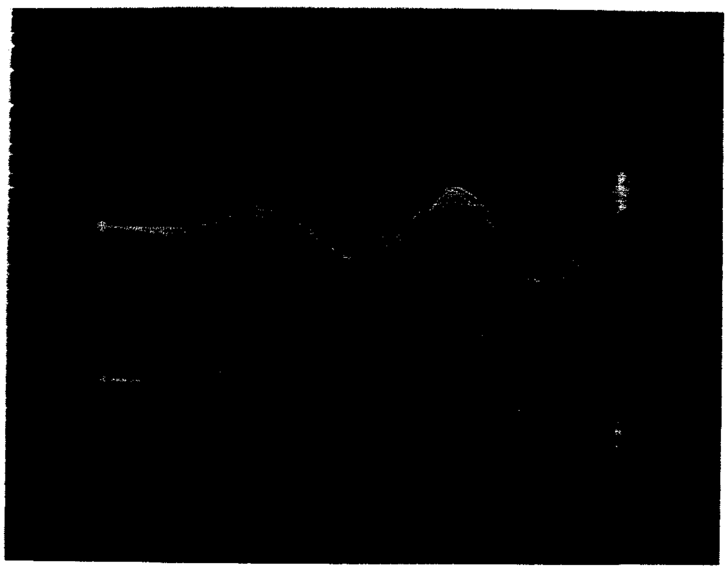
Noise was added to the probing signal and the plots of Figures 3-17 and 3-18 resulted. Note that signal-to-noise ratios less than 26 db result in highly uncertain measurements. The noise was inserted into both the nominal system and the tested system from independent generators. Because of equipment limitations, it was necessary to make all the system time constants ten times shorter so that the noise bandwidth would encompass the signal frequencies. This, of course, was only a change in the time base so that the results are entirely compatible with all others in this report. The signal-to-noise ratio was measured by the ratio

$$\frac{\text{peak rms voltage of the signal}}{\text{average rms noise voltage}}$$

Δd_o

S/N = 14 db

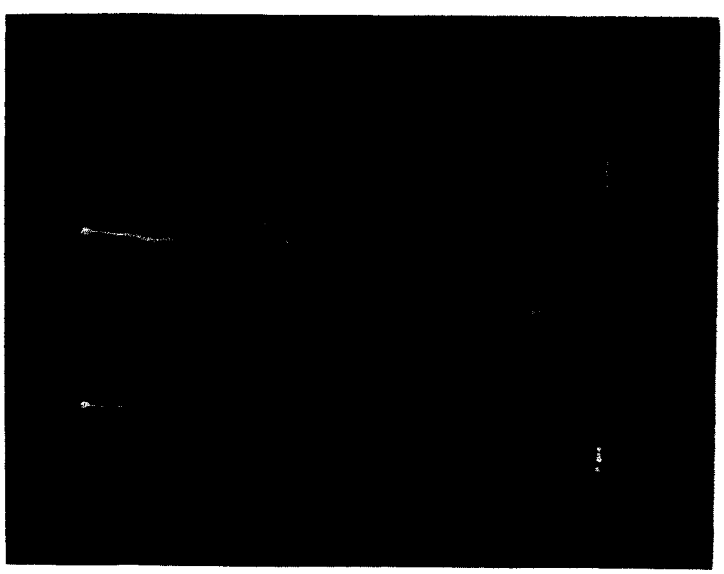
Δd_1



Δd_o

S/N = 20 db

Δd_1



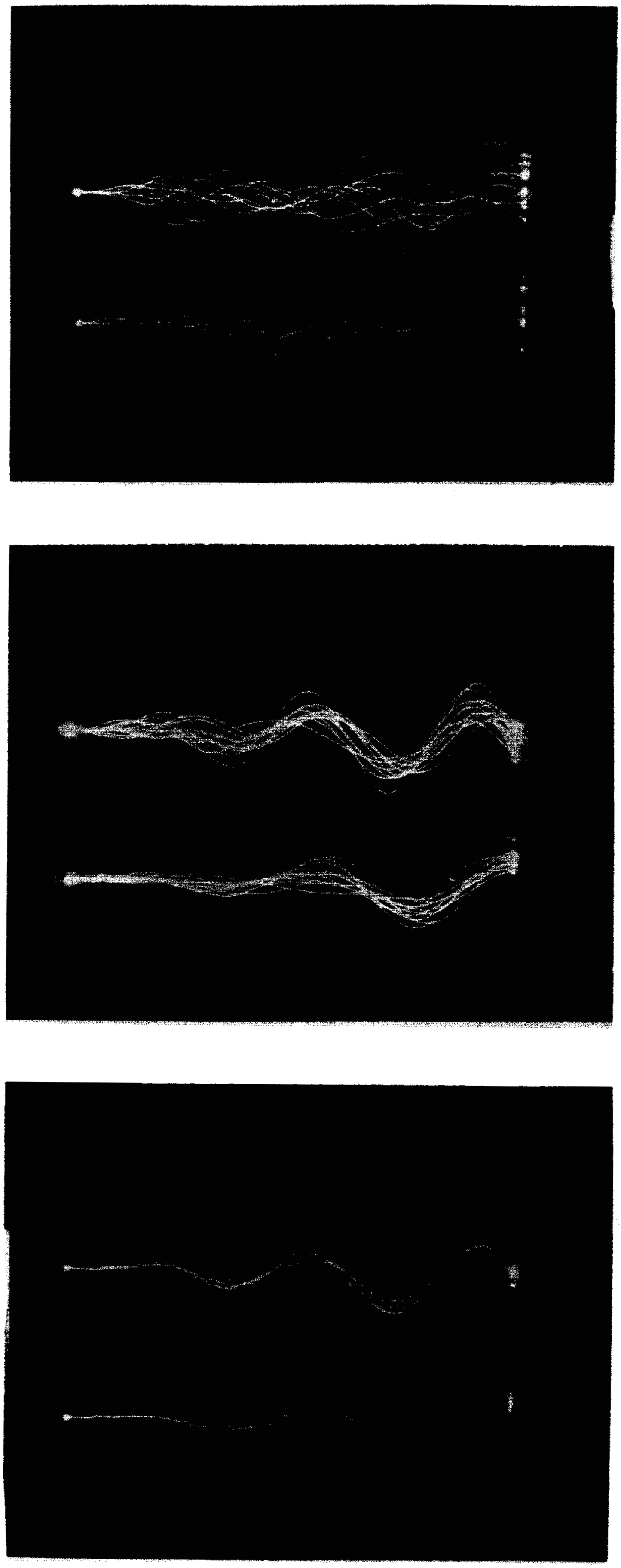
Δd_o

S/N = 26 db

Δd_1

FIGURE 3-17 Δd_0 = .10 FREQUENCY VARIATION

NOISE ENVIRONMENT



Δd_o

S/N = 14 db

Δd_1

Δd_o

S/N = 20 db

Δd_1

Δd_o

S/N = 26 db

Δd_1

FIGURE 3-18 Δd_1 = .10 DAMPING VARIATION

NOISE ENVIRONMENT

SECTION 4

SYSTEM CONFIDENCE SAMPLING

4.1 SUMMARY

A method of measuring the confidence of a transfer function has been established. The method gives value to and compares all the parameters of the transfer function that are unchanged if the value of one sample taken at a particular time is unchanged. The procedure requires relatively small amounts of equipment, when compared to the growing exponential method, and is easily adapted to any transfer function. The time for measurement is approximately 5 times the time constants of the circuit (i.e., the same as the growing exponential method). The method can not, however, synthesize what parameter is bad without providing a series of independent probing signals to establish what parameter has changed.

4.2 INTRODUCTION

In the Phase A report of this study, a limited introduction to the concepts of optimization were discussed. In this discussion it was pointed out that the testing of system performance by feedback control has a potential possibility of providing rapid test of operation with relatively small amounts of equipment, when compared to the method of growing exponentials. Because of this potential savings in equipment, the procedure was investigated further.

In the growing exponential theory, the impulse response of the partial system convoluted with the input signal gives the functional

$$h_{ji}(\tau) = \int_{-\infty}^{+\infty} h_j(t)\ f_i\ (\tau - t)\ dt$$

(Note: This is Eq. 3-27, Phase A Report SIMAR 59/2)

where $h_j(t)$ is the weighting function of a filter and $f_i(t)$ is the i-th input signal.

Letting $\tau = 0$ the equation gives the value of a sample of the output signal at $\tau = 0$ thus, obtaing

$$h_{jk} = \int_{-\infty}^{+\infty} h_j(t)\ f_i(t)\ dt.$$

Let us propose a question. What would happen if the impulse response of the desired system reversed in time was convoluted with the weighting function (impulse response) of the actual system being tested? If the results are sampled at time $\tau = 0$, then the value of the sampe will be

$$\int_{-\infty}^{+\infty} f_i^2(t)\ dt$$

if the parameters of the system are unchanged.

If the system has changed parameters, then the value of the sample will be different. The only exception will occur if both the shape and the size of the weighting function change in proper amounts to nullify the changes.

Now suppose a signal is applied and forced to be orthogonal to the impulse response of the desired system over negative time. The results of this resultant convolution integral sampled at $\tau = 0$ will be essentially zero. If the system has changed parameters, the sample may not be zero. The one condition which would allow the sample to equal zero would be a change in gain in the measured system, with no change in shape of the weighting function. Therefore, a test that could be performed on a system is to provide as a signal the negative time impulse response to the system and measure the change in the response of the system at a predetermined time. This would allow specification of of a Go-No Go system test which would give an indication that all of the parameter are within some limits.

4.3 NEGATIVE TIME IMPULSE RESPONSE

The negative time response of a network can be obtained from the inverse Laplace transform of the transfer function by replacing the operator (s) by (-s).

$$f(t) = L^{-1} \left[H(-s) \right]$$

The generation of this function can be put into a generalized form for any transfer function. This generalized form was discovered by the investigation of optimum feedback control. While the derivation is complicated, the results are intutive. Any transfer function can be represented by the following diagram as reported in the Phase I report.

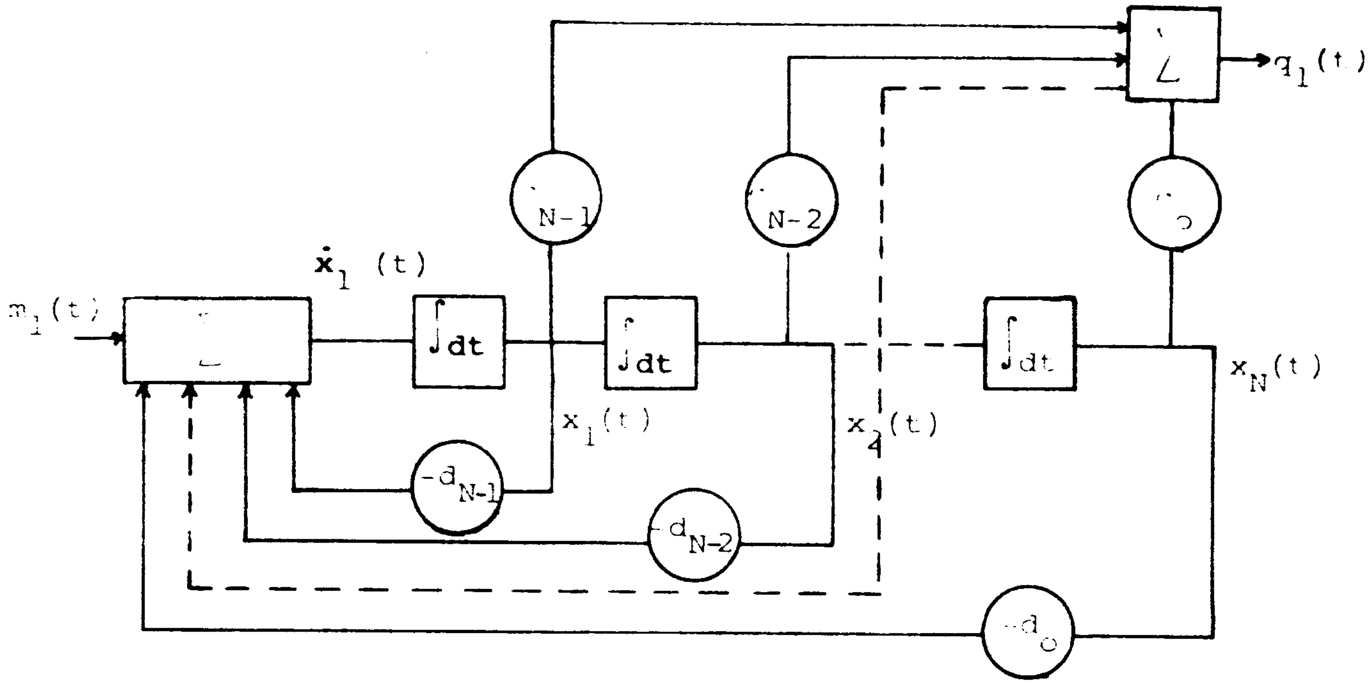


Figure 4-1

Analog Computer Simulation of Any Transfer Function

To generate the impulse response function reversed in time refer to the diagram in Figure 4-2

The only difference in the two diagrams is that the signs of the coefficient terms alternate. For example, if the transfer function is

$$H(s) = \frac{c_1 s + c_o}{s^2 + d_1 s + d_o}$$

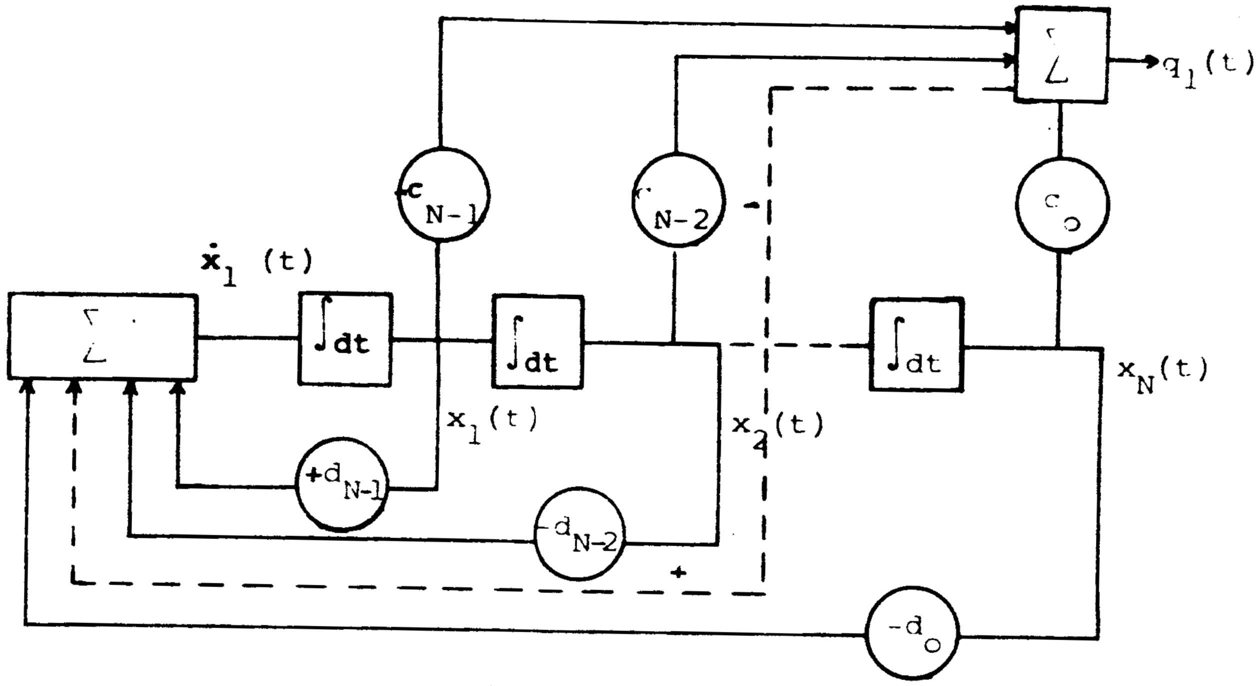


Figure 4-2

Analog Computer Simulation of Any
Impulse Generation

then

$$H(-s) = \frac{-c_1 s + c_o}{s^2 - d_1(s) + d_o}$$

Thus, giving alternating signs in the diagram. The results say that the negative time impulse function for any transfer function can be generated simply, and with very little equipment. If the transfer function is of the forms

$$\frac{c_1 s}{s^2 + d_1 s + d_o} \quad .$$

$$\frac{c_o}{s + d_o}$$

then simple positive feedback generates the negative time impulse response function.

These signals then can be fed into the system under test, and a sample taken at time $\tau = 0$. If a signal independent to the impulse function is desired, the input to the first integrators in the diagram can be used as an input signal, i.e., $\dot{X}_1(t)$.

$$\dot{X}_1(t) = + d_{N-1} X_2(t) - d_{N-2} X_3(t) + \ldots\ldots - d_o X_N(t)$$

This signal will be independent of the imulse response q(t), where: $q(t) = -c_{N-1} X_2(t) + c_{N-2} X_3(t) - \ldots\ldots c_o X_i(t)$, and if supplied to the system under test would give another independent check of whether the system's parameter were bad or good when sampled at a particular time, $\tau = 0$. In some particular cases, a signal orthogonal to the impulse function over negative time may be supplied to the system to be tested. These particular cases are; when there is only one term in the numerator of the transfer function. For example,

$$\frac{c_o}{s + d_o}$$

$$\frac{c_1}{s^2 + s\, d_1 + d_o}$$

$$\frac{c_2\, s^2}{s^3 + s^2\, d_2 + d_1 s + d_o} \qquad \text{etc.}$$

In these cases the output of any integrator in the diagram will supply an orthogonal signal to the impulse function overall negative time.

To test which parameter has changed several independent, signals would have to be provided each designed to measure one or more parameters. The testing time for this procedure would approach sinusoidal testing. These independent signals could be obtained by setting various sets of coefficients c_i to zero. An experimental program could establish the measuring sensitivities of the particular parameters with respect to the coefficient c_i. Thus, a particular set of probing functions could possibly be found which would measure a set of parameters. This method is not recommended for parameter measurement since the growing exponential method performs the measurement of several parameters with one probing signals.

The main importances and uses of the method discussed here are:

a. Confidence checking of the system parameters.

b. Equipment requirements are less than the growing exponential method.

c. This test saves time in measuring for confidence checking.

Where are we going? The Phase C effort will be pointed in the direction of a simulated system with a linear system transfer function associated with a Saturn Subsystem. The objective will aim at answering the pertinent questions posed above. Other established applicable techniques and questions will be extended within the time and money available.

SECTION 5

CONCLUSIONS AND RECOMMENDATIONS

1. It has been demonstrated that linear active networks can be interpreted as linear system transfer functions with limited input signal amplitudes.

2. Experimental results clearly demonstrate the ability to measure variations in coefficients of general first and second order transfer functions.

3. Non-linear properties in coefficients of active networks may be ignored if the signal amplitude can be limited to the linear region.

4. Simultaneous variations in many parameters may require longer testing routines to establish the values of each parameter variation.

5. Noisy signal degradation in the measuring process is independent of the complexity of the system under investigation. This is illustrated by the 26 db S/N for first and second order systems.

6. Absolute measurement accuracy is a function of system complexity, as illustrated by the contour in Figure 3-12.

7. Complicated transfer functions can be reduced by a difference technique or a compensation technique.

RECOMMENDATIONS

The technique of utilizing growing exponential input probing signals must be evaluated on a real system and compared on a competitive basis with existing checkout methods.

BIBLIOGRAPHY

1. "Single Parameter Testing"
   Phase A Completion Report
   64 SIMAR 59/2

2. "Transistors and Active Circuits"
   by Linvill, and Gibbons
   McGraw Hill, 1961

3. "Network Analysis and Feedback Amplifier Design"
   H.W. Bode
   D. VanNostrand Co., Inc., 6th printing, 1959

4. "Control System Synthesis"
   J. Truxal
   McGraw Hill, 1955

5. "The Fourier Integral and Its Applications"
   A. Papoulis
   McGraw Hill, 1962